

thm_2Eint__arith_2ENOT__INT__DIVIDES__POS (TMUxqVJzU8M9hrw6FJkEbyGqxwUodHySp3m)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{3}$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{ty_2Einteger_2Eint}) \tag{4}$$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 6 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty_2Einteger_2Eint_REP_CLASS$

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)_{ty_2Einteger_2Etint_add})_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)} \tag{5}$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (7)$$

Definition 7 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 8 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (8)$$

Definition 9 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint_neg)$

Definition 10 We define $c_2Einteger_2Eint_sub$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (10)$$

Definition 11 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 12 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (12)$$

Definition 13 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (13)$$

Definition 14 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge (p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0c \in ty_2Einteger_2Eint.(\forall V1d \in ty_2Einteger_2Eint. \\ & ((\neg(V0c = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))) \Rightarrow ((\neg \\ & (p\ (ap\ (ap\ c_2Einteger_2Eint_divides\ V0c)\ V1d))) \Leftrightarrow (\exists V2i \in \\ & ty_2Einteger_2Eint.((p\ (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ c_2Einteger_2Eint_of_num \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \\ & V2i)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Eint_le\ V2i)\ (ap\ (ap\ c_2Einteger_2Eint_sub \\ & (ap\ c_2Einteger_2EABS\ V0c))\ (ap\ c_2Einteger_2Eint_of_num\ (\\ & ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \wedge \\ & (p\ (ap\ (ap\ c_2Einteger_2Eint_divides\ V0c)\ (ap\ (ap\ c_2Einteger_2Eint_add \\ & V1d)\ V2i)))))))))) \end{aligned} \quad (23)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap\ c_2Einteger_2Eint_of_num\ V0m) = (ap\ c_2Einteger_2Eint_of_num\ V1n)) \Leftrightarrow (V0m = V1n)))) \quad (24)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap\ c_2Einteger_2EABS\ (ap\ c_2Einteger_2Eint_of_num\ V0n)) = (ap\ c_2Einteger_2Eint_of_num\ V0n))) \quad (25)$$

Theorem 1

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(\forall V1d \in ty_2Einteger_2Eint. \\ & ((\neg(V0n = c_2Enum_2E0)) \Rightarrow ((\neg(p\ (ap\ (ap\ c_2Einteger_2Eint_divides \\ & (ap\ c_2Einteger_2Eint_of_num\ V0n))\ V1d))) \Leftrightarrow (\exists V2i \in ty_2Einteger_2Eint. \\ & (((p\ (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ c_2Einteger_2Eint_of_num \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \\ & V2i)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Eint_le\ V2i)\ (ap\ (ap\ c_2Einteger_2Eint_sub \\ & (ap\ c_2Einteger_2Eint_of_num\ V0n))\ (ap\ c_2Einteger_2Eint_of_num \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \wedge \\ & (p\ (ap\ (ap\ c_2Einteger_2Eint_divides\ (ap\ c_2Einteger_2Eint_of_num\ V0n))\ (ap\ (ap\ c_2Einteger_2Eint_add\ V1d)\ V2i)))))))))) \end{aligned}$$