

thm_2Eint_arith_2Ecan_get_small
(TMUDgzXvLX7EMmDkKfpn5mbL76xc7pYxW4y)

October 26, 2020

Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2.V2t))))$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p (\text{ap } P x)))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2ECOND` to be $\lambda A. 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 27a. (\lambda V2t2 \in A. 27a. (\text{ap } (\text{c_2Emin_2E_40 } (2^{A-27a}))))))$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \tag{1}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 A1) \tag{2}$$

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Einteger_2Eint} \tag{3}$$

Let `c_2Einteger_2Eint_REP_CLASS` : ι be given. Assume the following.

$$\text{c_2Einteger_2Eint_REP_CLASS} \in ((2^{(\text{ty_2Epair_2Eprod } \text{ty_2Enum_2Enum } \text{ty_2Enum_2Enum}) \text{ty_2Einteger_2Eint}})) \tag{4}$$

Definition 9 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E40 (ty_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (5)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}} \quad (7)$$

Definition 10 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$

Definition 11 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (8)$$

Definition 12 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint$

Definition 13 We define $c_2Einteger_2Eint_sub$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$

Definition 14 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E40$

Let $c_2Einteger_2Eint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (9)$$

Definition 15 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 16 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E3D_3D_3E V0t) c_2Ebool_2E7E$

Definition 17 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$

Let $c_2Einteger_2Eint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (10)$$

Definition 18 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (14)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (15)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (16)$$

Definition 19 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 20 We define $c_2Eprim_rec_2E.3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 21 We define $c_2Earithmetic_2E.3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 22 We define $c_2Ebool_2E.5C.2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E.21\ 2)\ (\lambda V2t \in$

Definition 23 We define $c_2Earithmetic_2E.3E.3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (17)$$

Definition 24 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 25 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2E$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Let $c_2Earithmetic_2E.2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E.2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Earithmetic_2E.2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E.2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Definition 26 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 27 We define $c_2Enumeral_2EiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Enum_2ESUC\ (ap$

Let $c_2Earithmic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (21)$$

Definition 28 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 29 We define $c_2Earithmic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmic_2E_2B$

Definition 30 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmic_2E_2B$

Definition 31 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Definition 32 We define $c_2Earithmic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Assume the following.

$$((ap\ c_2Earithmic_2ENUMERAL\ (ap\ c_2Earithmic_2EBIT2\ c_2Earithmic_2EZERO)) = (ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmic_2ENUMERAL\ (ap\ c_2Earithmic_2EBIT1\ c_2Earithmic_2EZERO)))) \quad (22)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n)) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Enum_2ESUC\ V0m))\ V1n)))) \quad (23)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ c_2Enum_2E0)\ V0n))) \quad (24)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ V1n)\ V0m)))) \quad (25)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V0m) \\
& (ap (ap c_2Earithmetic_2E_2A V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2A \\
& (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V2p))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
& c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) V0n)))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2A \\
& (ap c_2Enum_2ESUC V2p)) V0m)) (ap (ap c_2Earithmetic_2E_2A (ap \\
& c_2Enum_2ESUC V2p)) V1n))))))
\end{aligned} \tag{31}$$

Assume the following.

$$True \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))) \quad (42)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (43)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (44)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (45)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (46)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))) \quad (47)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\exists V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p (ap V0P V2x))))) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p V0P) \vee (\exists V2x \in A_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A_27a.((p V0P) \vee (p (ap V1Q V3x)))))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ((p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (53)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (54)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (55)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\ & \quad \forall V0P \in ((2^{A.27b})^{A.27a}).((\forall V1x \in A.27a.(\exists V2y \in \\ & A.27b.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}).(\\ & \quad \forall V4x \in A.27a.(p (ap (ap V0P V4x) (ap V3f V4x)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Einteger_2Eint.(\forall V1x \in ty_2Einteger_2Eint. \\ & (\forall V2y \in ty_2Einteger_2Eint.((p (ap (ap c_2Einteger_2Eint_lt \\ & (ap c_2Einteger_2Eint_of_num c_2Enum_2E0) V0n)) \Rightarrow ((p (ap (\\ & ap c_2Einteger_2Eint_lt V1x) V2y)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt \\ & (ap (ap c_2Einteger_2Eint_mul V0n) V1x)) (ap (ap c_2Einteger_2Eint_mul \\ & V0n) V2y)))))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. \\ & (\forall V2z \in ty_2Einteger_2Eint.((ap (ap c_2Einteger_2Eint_sub \\ & V0x) (ap (ap c_2Einteger_2Eint_sub V1y) V2z)) = (ap (ap c_2Einteger_2Eint_sub \\ & (ap (ap c_2Einteger_2Eint_add V0x) V2z)) V1y)))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0c \in ty_2Einteger_2Eint.(\forall V1b \in ty_2Einteger_2Eint. \\ & (\forall V2a \in ty_2Einteger_2Eint.((ap (ap c_2Einteger_2Eint_add \\ & (ap (ap c_2Einteger_2Eint_sub V2a) V0c)) V1b) = (ap (ap c_2Einteger_2Eint_sub \\ & (ap (ap c_2Einteger_2Eint_add V2a) V1b)) V0c)))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0y \in ty_2Einteger_2Eint.(\forall V1x \in ty_2Einteger_2Eint. \\ & ((ap (ap c_2Einteger_2Eint_mul V1x) V0y) = (ap (ap c_2Einteger_2Eint_mul \\ & V0y) V1x)))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
V2x) (ap (ap c_2Einteger_2Eint_mul V1y) V0z)) = (ap (ap c_2Einteger_2Eint_mul \\
& (ap (ap c_2Einteger_2Eint_mul V2x) V1y)) V0z))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((V0x = V1y) \vee ((p (ap (ap c_2Einteger_2Eint_lt V0x) V1y)) \vee (p (ap \\
& (ap c_2Einteger_2Eint_lt V1y) V0x))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. (((p (ap (ap c_2Einteger_2Eint_lt \\
V0x) V1y)) \wedge (p (ap (ap c_2Einteger_2Eint_lt V1y) V2z))) \Rightarrow (p (ap \\
& (ap c_2Einteger_2Eint_lt V0x) V2z))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
V0x) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) = V0x))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_mul V0x) \\
V1y)) = (ap (ap c_2Einteger_2Eint_mul V0x) (ap c_2Einteger_2Eint_neg \\
& V1y))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((p (ap (ap c_2Einteger_2Eint_le V0x) V1y)) \Leftrightarrow ((p (ap (ap c_2Einteger_2Eint_lt \\
V0x) V1y)) \vee (V0x = V1y))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_sub V0x) \\
V1y)) = (ap (ap c_2Einteger_2Eint_sub V1y) V0x))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_sub V0x) V1y))) \Leftrightarrow (p (\\
& ap (ap c_2Einteger_2Eint_lt V1y) V0x))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& V0x) (ap (ap c_2Einteger_2Eint_sub V1y) V2z)) = (ap (ap c_2Einteger_2Eint_sub \\
& (ap (ap c_2Einteger_2Eint_mul V0x) V1y)) (ap (ap c_2Einteger_2Eint_mul \\
& V0x) V2z))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& (ap (ap c_2Einteger_2Eint_sub V0x) V1y)) V2z) = (ap (ap c_2Einteger_2Eint_sub \\
& (ap (ap c_2Einteger_2Eint_mul V0x) V2z)) (ap (ap c_2Einteger_2Eint_mul \\
& V1y) V2z))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((p (ap (ap c_2Einteger_2Eint_lt V0x) (ap (ap c_2Einteger_2Eint_add \\
& V0x) V1y))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V1y))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& V0m)) (ap c_2Einteger_2Eint_of_num V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V0m) V1n))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_of_num \\
& V0m)) (ap c_2Einteger_2Eint_of_num V1n)) = (ap c_2Einteger_2Eint_of_num \\
& (ap (ap c_2Earithmetric_2E_2A V0m) V1n))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_lt \\
& (ap (ap c_2Einteger_2Eint_add V0x) V1y)) V2z)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt \\
& V0x) (ap (ap c_2Einteger_2Eint_sub V2z) V1y))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. ((p (ap (ap (ap c_2Einteger_2Eint_lt \\
& (ap (ap c_2Einteger_2Eint_sub V0x) V1y)) V2z)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt \\
& V0x) (ap (ap c_2Einteger_2Eint_add V2z) V1y)))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\
& V0x)) (ap c_2Einteger_2Eint_neg V1y))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt \\
& V1y) V0x))))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& V0n)) (ap c_2Einteger_2Eint_of_num V1m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V0n)) (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V1m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V1m) V0n))) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num V0n)) (ap c_2Einteger_2Eint_of_num \\
& V1m))) \Leftrightarrow ((\neg(V0n = c_2Enum_2E0)) \vee (\neg(V1m = c_2Enum_2E0)))) \wedge ((p \\
& (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& V0n)) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& V1m)))) \Leftrightarrow False))))))
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& (\forall V0i \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_le \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0i)) \Rightarrow (\exists V1n \in \\
& ty_2Enum_2Enum. (V0i = (ap c_2Einteger_2Eint_of_num V1n))))))
\end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Einteger_2Eint. (\forall V1q \in ty_2Einteger_2Eint. \\
& (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_mul V0p) V1q))) \Leftrightarrow (((\\
& p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V0p)) \wedge (p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V1q))) \vee ((p (ap (ap c_2Einteger_2Eint_lt V0p) (\\
& ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \wedge (p (ap (ap c_2Einteger_2Eint_lt \\
& V1q) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)))))) \wedge ((p (\\
& ap (ap c_2Einteger_2Eint_lt (ap (ap c_2Einteger_2Eint_mul V0p) \\
& V1q)) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow (((p (ap \\
& (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \\
& V0p)) \wedge (p (ap (ap c_2Einteger_2Eint_lt V1q) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)))))) \vee ((p (ap (ap c_2Einteger_2Eint_lt V0p) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))) \wedge (p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V1q)))))))))
\end{aligned}$$

(80)

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& (ap c_2Earithmetic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg(p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n)))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{84}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0n) c_2Enum_2E0)))) \tag{85}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{86}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{87}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{88}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (89)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (90)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (91)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (92)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (93)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (94)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (95)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (96)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (97)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (98)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (99)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (100)$$

Theorem 1

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint.(\forall V2d \in ty_2Einteger_2Eint.((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V2d)) \Rightarrow (\exists V3c \in ty_2Einteger_2Eint.((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V3c)) \wedge (p (ap (ap c_2Einteger_2Eint_lt (ap (ap c_2Einteger_2Eint_sub V1y) (ap (ap c_2Einteger_2Eint_mul V3c) V2d))) V0x))))))))))$$