

thm_2Eint_arith_2Eelim_le_coeffs (TM-FGPfP6cFLHtF3LU95kCFjTNJhmUS9oQB9)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow & nonempty\ (ty_2Epair_2Eprod \\ & A0\ A1) \end{aligned} \quad (2)$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (3)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (4)$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint. (ap (c_2Emin_2E_40 (ty_2Einteger_2Eint$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (5)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)} \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}} \quad (7)$$

Definition 6 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 7 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint_ABS\ T1)$

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum))^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)} \quad (8)$$

Definition 8 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Etint_add\ T1\ T2)$

Definition 9 We define $c_2Einteger_2Eint_sub$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Etint_add\ (\lambda V0x\ V1y)\ (\lambda V0x\ V1y))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (9)$$

Let $c_2Enum_2EAbs_num : \iota$ be given. Assume the following.

$$c_2Enum_2EAbs_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EAbs_num\ c_2Enum_2EZERO_REP)$.

Definition 11 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (12)$$

Definition 12 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EAbs_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (13)$$

Definition 13 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n)\ (\lambda V0n\ V1m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EAbs_num\ m)))$

Definition 14 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 15 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 16 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 17 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 18 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Let $c_2Einteger_2Eint_mod : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_mod \in ((ty_2Einteger_2Eint^{ty_2Einteger_2Eint})^{ty_2Einteger_2Eint}) \quad (14)$$

Let $c_2Einteger_2Eint_div : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_div \in ((ty_2Einteger_2Eint^{ty_2Einteger_2Eint})^{ty_2Einteger_2Eint}) \quad (15)$$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{\dots})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (16)$$

Definition 19 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. \lambda V1T2 \in ty_2Einteger_2Eint. V0T1 * V1T2$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (17)$$

Definition 20 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. \lambda V1T2 \in ty_2Einteger_2Eint. V0T1 < V1T2$

Definition 21 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 22 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint. \lambda V1y \in ty_2Einteger_2Eint. V0x \leq V1y$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (18)$$

Definition 23 We define $c_2Emarker_2EAbbrev$ to be $\lambda V0x \in 2.V0x$.

Definition 24 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (27)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (28)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (29)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (\forall V0t1 \in A_27a.(\forall V1t2 \in \\ A_27a.((ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ V1t2) = V0t1) \wedge ((ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\ V0t1) V1t2) = V1t2))) \end{aligned} \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B)) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap (ap (c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & V5y_27))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
 & ((p (ap (ap c_2Einteger_2Eint_lt V0x) V1y)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le \\
 & V0x) (ap (ap c_2Einteger_2Eint_add V1y) (ap c_2Einteger_2Eint_neg \\
 & (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))))))) \\
 \end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_sub \\
 & (ap (ap c_2Einteger_2Eint_add V0x) (ap c_2Einteger_2Eint_of_num \\
 & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \\
 & (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0x)) \\
 \end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0y \in ty_2Einteger_2Eint. (\forall V1x \in ty_2Einteger_2Eint. \\
 & ((ap (ap c_2Einteger_2Eint_mul V1x) V0y) = (ap (ap c_2Einteger_2Eint_mul \\
 & V0y) V1x)))) \\
 \end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0z \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
 & (\forall V2x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add \\
 & V2x) (ap (ap c_2Einteger_2Eint_add V1y) V0z)) = (ap (ap c_2Einteger_2Eint_add \\
 & (ap (ap c_2Einteger_2Eint_add V2x) V1y)) V0z)))))) \\
 \end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0z \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
 & (\forall V2x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
 & V2x) (ap (ap c_2Einteger_2Eint_add V1y) V0z)) = (ap (ap c_2Einteger_2Eint_add \\
 & (ap (ap c_2Einteger_2Eint_mul V2x) V1y)) (ap (ap c_2Einteger_2Eint_mul \\
 & V2x) V0z)))))) \\
 \end{aligned} \tag{43}$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint. (\neg(p (ap (ap c_2Einteger_2Eint_lt \\
 V0x) V0x)))) \tag{44}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
 & V0x) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0x)) \\
 \end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
 & V0x) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num \\
 & c_2Enum_2E0))) \\
 \end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
 & (\forall V2z \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_lt \\
 & (ap (ap c_2Einteger_2Eint_add V0x) V1y)) (ap (ap c_2Einteger_2Eint_add \\
 & V0x) V2z))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt V1y) V2z))))))) \\
 \end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
 & (\neg((p (ap (ap c_2Einteger_2Eint_lt V0x) V1y)) \wedge (p (ap (ap c_2Einteger_2Eint_lt \\
 & V1y) V0x)))))) \\
 \end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
 & (\forall V2z \in ty_2Einteger_2Eint. (((p (ap (ap c_2Einteger_2Eint_le \\
 & V0x) V1y)) \wedge (p (ap (ap c_2Einteger_2Eint_lt V1y) V2z))) \Rightarrow (p (ap \\
 & (ap c_2Einteger_2Eint_lt V0x) V2z)))))) \\
 \end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
 & (\forall V2z \in ty_2Einteger_2Eint. (((p (ap (ap c_2Einteger_2Eint_le \\
 & V0x) V1y)) \wedge (p (ap (ap c_2Einteger_2Eint_le V1y) V2z))) \Rightarrow (p (ap \\
 & (ap c_2Einteger_2Eint_le V0x) V2z)))))) \\
 \end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
 & ((p (ap (ap c_2Einteger_2Eint_le V0x) (ap (ap c_2Einteger_2Eint_add \\
 & V0x) V1y))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num \\
 & c_2Enum_2E0)) V1y)))))) \\
 \end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0q \in ty_2Einteger_2Eint. ((\neg(V0q = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \Rightarrow (\forall V1p \in ty_2Einteger_2Eint. ((V1p = (ap (ap c_2Einteger_2Eint_add (ap (ap c_2Einteger_2Eint_mul (ap (ap c_2Einteger_2Eint_div V1p) V0q)) (ap (ap c_2Einteger_2Eint_mod V1p) V0q)))) \wedge (p (ap (ap (c_2Ebool_2ECOND 2) (ap (ap c_2Einteger_2Eint_lt V0q) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) (ap (ap c_2Ebool_2E_2F_5C (ap (ap c_2Einteger_2Eint_lt V0q) (ap (ap c_2Einteger_2Eint_mod V1p) V0q))) (ap (ap c_2Einteger_2Eint_le (ap (ap c_2Einteger_2Eint_mod V1p) V0q))) (ap (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)))) (ap (ap c_2Ebool_2E_2F_5C (ap (ap c_2Einteger_2Eint_le (ap (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) (ap (ap c_2Einteger_2Eint_mod V1p) V0q))) (ap (ap c_2Einteger_2Eint_lt (ap (ap c_2Einteger_2Eint_mod V1p) V0q)))))))))) \\
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0x)) \Rightarrow ((p (ap (ap c_2Einteger_2Eint_lt (ap (ap c_2Einteger_2Eint_mul V0x) V1y)) (ap (ap c_2Einteger_2Eint_mul V0x) V2z)))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt V1y) V2z))))))) \\
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0x)) \Rightarrow ((p (ap (ap c_2Einteger_2Eint_lt (ap (ap c_2Einteger_2Eint_mul V0x) V1y)) (ap (ap c_2Einteger_2Eint_mul V0x) V2z)))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt V1y) V2z))))))) \\
\end{aligned} \tag{54}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{55}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
\end{aligned} \tag{58}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V1q) \vee (p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & ((\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & (\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (63)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (69)$$

Theorem 1

$$\begin{aligned} & (\forall V0m \in ty_2Einteger_2Eint. (\forall V1n \in ty_2Einteger_2Eint. \\ & (\forall V2x \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_lt \\ & (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0m)) \Rightarrow ((p (ap (\\ & ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \\ & (ap (ap c_2Einteger_2Eint_add (ap (ap c_2Einteger_2Eint_mul \\ & V0m) V2x)) V1n))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num \\ & c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add V2x) (ap (ap c_2Einteger_2Eint_div \\ & V1n) V0m)))))))) \end{aligned}$$