

thm\_2Eint\_arith\_2Egcd21\_thm  
 (TMLrK1nhP9axrBiFwHNZVkkWT1gikv2jjP2)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota))$

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c\_2Emin\_2E\_40 A) a)))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Egcd\_2Egcd : \iota$  be given. Assume the following.

$$c\_2Egcd\_2Egcd \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (2)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow & nonempty\ (ty\_2Epair\_2Eprod \\ & A0\ A1) \end{aligned} \quad (3)$$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \quad (4)$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint}) \quad (5)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) a)))$

**Definition 6** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint. (ap (c\_2Emin\_2E\_40 (ty\_2Einteger\_2Eint)))$

Let  $c\_2Einteger\_2Etint\_mul : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_mul \in (((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (6)$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (7)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)}} \quad (8)$$

**Definition 7** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)$

**Definition 8** We define  $c\_2Einteger\_2Eint\_mul$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint. \lambda V1T2 \in ty\_2Einteger\_2Eint$

Let  $c\_2Einteger\_2Etint\_add : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_add \in (((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (9)$$

**Definition 9** We define  $c\_2Einteger\_2Eint\_add$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint. \lambda V1T2 \in ty\_2Einteger\_2Eint$

Let  $c\_2Einteger\_2Eint\_div : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_div \in ((ty\_2Einteger\_2Eint)^{ty\_2Einteger\_2Eint})^{ty\_2Einteger\_2Eint} \quad (10)$$

Let  $c\_2Einteger\_2Etint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_neg \in ((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (11)$$

**Definition 10** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint. (ap c\_2Einteger\_2Eint)$

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint)^{ty\_2Enum\_2Enum} \quad (12)$$

**Definition 11** We define  $c\_2Einteger\_2Eint\_divides$  to be  $\lambda V0p \in ty\_2Einteger\_2Eint. \lambda V1q \in ty\_2Einteger\_2Eint$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (13)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (14)$$

**Definition 12** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 13** We define  $c_{\text{Emin}} : \text{D}_3 \rightarrow \text{D}_3$  to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o} (p \Rightarrow p \quad Q)$  of type  $\iota$ .

**Definition 14** We define  $c_{\text{Ebool\_2E\_7E}}$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c_{\text{2Emin\_2E\_3D\_3D\_3E}}\ V0t)\ c_{\text{2Ebool\_2E\_7E}}))$

**Definition 15** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Let  $c\_2Enum\_2EREPE\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (15)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^\omega) \quad (16)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (17)$$

**Definition 16** We define  $c\_2Enum\_2EESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m\ 0)$

**Definition 17** We define  $c_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 18** We define  $c_{\cdot 2Earthmetic\_2E\_3E}$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 19** We define  $c_{\text{C\_Ebool\_2E\_5C\_2F}}$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_{\text{C\_Ebool\_2E\_21}}) 2)) (\lambda V2t \in$

**Definition 20** We define  $c_{\text{Earthmet}} : \lambda V0m \in \text{ty\_Enum} . \lambda V1n \in \text{ty\_Enum} . \lambda V2o \in \text{ty\_Enum} . \lambda V3p \in \text{ty\_Enum} . \lambda V4q \in \text{ty\_Enum} . \lambda V5r \in \text{ty\_Enum} . \lambda V6s \in \text{ty\_Enum} . \lambda V7t \in \text{ty\_Enum} . \lambda V8u \in \text{ty\_Enum} . \lambda V9v \in \text{ty\_Enum} . \lambda V10w \in \text{ty\_Enum} . \lambda V11x \in \text{ty\_Enum} . \lambda V12y \in \text{ty\_Enum} . \lambda V13z \in \text{ty\_Enum}$

Let  $c\_2Enum\_2ZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ZERO\_REP \in \omega \quad (18)$$

**Definition 21** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 22** We define  $c_{\_2Ebool\_2ECOND}$  to be  $\lambda A.\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.\_27a.(\lambda V2t2 \in A.\_27a.($

**Definition 23** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (ap (c\_2Ebool\_2B$

Let  $c_2$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \\ (19)$$

Let  $c_2Earithmetic_2E_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (20)$$

Let  $c_2$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (21)$$

**Definition 24** We define  $c\_2\text{Enumeral\_2EiZ}$  to be  $\lambda V0x \in ty\_2\text{Enum\_2Enum}.V0x$ .

**Definition 25** We define  $c\_2\text{Earithmetic\_2ENUMERAL}$  to be  $\lambda V0x \in ty\_2\text{Enum\_2Enum}.V0x$ .

Let  $c\_2\text{Earithmetic\_2E\_2B} : \iota$  be given. Assume the following.

$$c\_2\text{Earithmetic\_2E\_2B} \in ((ty\_2\text{Enum\_2Enum}^{ty\_2\text{Enum\_2Enum}})^{ty\_2\text{Enum\_2Enum}}) \quad (22)$$

**Definition 26** We define  $c\_2\text{Earithmetic\_2EBIT2}$  to be  $\lambda V0n \in ty\_2\text{Enum\_2Enum}.(ap (ap c\_2\text{Earithmetic\_2E\_2B} V0n))$

**Definition 27** We define  $c\_2\text{Earithmetic\_2EBIT1}$  to be  $\lambda V0n \in ty\_2\text{Enum\_2Enum}.(ap (ap c\_2\text{Earithmetic\_2E\_2B} V0n))$

**Definition 28** We define  $c\_2\text{Earithmetic\_2EZERO}$  to be  $c\_2\text{Enum\_2E0}$ .

**Definition 29** We define  $c\_2\text{Earithmetic\_2E\_3C\_3D}$  to be  $\lambda V0m \in ty\_2\text{Enum\_2Enum}.(\lambda V1n \in ty\_2\text{Enum\_2Enum}.(ap (ap c\_2\text{Earithmetic\_2E\_3C\_3D} c\_2\text{Enum\_2E0}) V0m)))$

Assume the following.

$$(\forall V0n \in ty\_2\text{Enum\_2Enum}.(p (ap (ap c\_2\text{Earithmetic\_2E\_3C\_3D} c\_2\text{Enum\_2E0}) V0n))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2\text{Enum\_2Enum}.(\forall V1n \in ty\_2\text{Enum\_2Enum}.( \\ & ((ap (ap c\_2\text{Earithmetic\_2E\_2A} c\_2\text{Enum\_2E0}) V0m) = c\_2\text{Enum\_2E0}) \wedge \\ & (((ap (ap c\_2\text{Earithmetic\_2E\_2A} V0m) c\_2\text{Enum\_2E0}) = c\_2\text{Enum\_2E0}) \wedge \\ & (((ap (ap c\_2\text{Earithmetic\_2E\_2A} (ap c\_2\text{Earithmetic\_2ENUMERAL} \\ & (ap c\_2\text{Earithmetic\_2EBIT1} c\_2\text{Earithmetic\_2EZERO}))) V0m) = V0m) \wedge \\ & (((ap (ap c\_2\text{Earithmetic\_2E\_2A} V0m) (ap c\_2\text{Earithmetic\_2ENUMERAL} \\ & (ap c\_2\text{Earithmetic\_2EBIT1} c\_2\text{Earithmetic\_2EZERO}))) = V0m) \wedge \\ & ((ap (ap c\_2\text{Earithmetic\_2E\_2A} (ap c\_2\text{Enum\_2ESUC} V0m)) V1n) = (ap \\ & (ap c\_2\text{Earithmetic\_2E\_2B} (ap (ap c\_2\text{Earithmetic\_2E\_2A} V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c\_2\text{Earithmetic\_2E\_2A} V0m) (ap c\_2\text{Enum\_2ESUC} V1n)) = \\ & (ap (ap c\_2\text{Earithmetic\_2E\_2B} V0m) (ap (ap c\_2\text{Earithmetic\_2E\_2A} \\ & V0m) V1n)))))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2\text{Enum\_2Enum}.(\forall V1n \in ty\_2\text{Enum\_2Enum}.( \\ & \forall V2p \in ty\_2\text{Enum\_2Enum}.(((p (ap (ap c\_2\text{Earithmetic\_2E\_3C\_3D} \\ & V0m) V1n)) \wedge (p (ap (ap c\_2\text{Earithmetic\_2E\_3C\_3D} V1n) V2p))) \Rightarrow (p ( \\ & ap (ap c\_2\text{Earithmetic\_2E\_3C\_3D} V0m) V2p))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2\text{Enum\_2Enum}.(\forall V1n \in ty\_2\text{Enum\_2Enum}.( \\ & (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2\text{Earithmetic\_2E\_3C\_3D} V0m) V1n)) \wedge (p ( \\ & ap (ap c\_2\text{Earithmetic\_2E\_3C\_3D} V1n) V0m))))))) \end{aligned} \quad (26)$$

Assume the following.

$$True \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (30)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t) \Leftrightarrow (p V0t))) \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. (((((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True)) \wedge (((False \vee (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2. (((((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True)) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (37)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (38)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (39)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (45)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (46)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow \quad (48) \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1a \in ty\_2Enum\_2Enum. (\forall V2x \in ty\_2Einteger\_2Eint. (\forall V3b \in ty\_2Einteger\_2Eint. (\forall V4d \in ty\_2Enum\_2Enum. (\forall V5p \in ty\_2Einteger\_2Eint. (\forall V6q \in ty\_2Einteger\_2Eint. (((V4d = (ap (ap c\_2Egcd\_2Egcd V1a) V0m)) \wedge (((ap c\_2Einteger\_2Eint\_of\_num V4d) = (ap (ap c\_2Einteger\_2Eint\_add (ap (ap c\_2Einteger\_2Eint\_mul V5p) (ap c\_2Einteger\_2Eint\_of\_num V1a))) (ap (ap c\_2Einteger\_2Eint\_mul V6q) (ap c\_2Einteger\_2Eint\_of\_num V0m)))) \wedge ((\neg(V4d = c\_2Enum\_2E0)) \wedge ((\neg(V0m = c\_2Enum\_2E0)) \wedge (\neg(V1a = c\_2Enum\_2E0)))))) \Rightarrow ((p (ap (ap c\_2Einteger\_2Eint\_divides (ap c\_2Einteger\_2Eint\_of\_num V0m)) (ap (ap c\_2Einteger\_2Eint\_add (ap (ap c\_2Einteger\_2Eint\_mul (ap c\_2Einteger\_2Eint\_of\_num V1a)) V2x)) V3b))) \Leftrightarrow ((p (ap (ap c\_2Einteger\_2Eint\_divides (ap c\_2Einteger\_2Eint\_of\_num V4d)) V3b)) \wedge (\exists V7t \in ty\_2Einteger\_2Eint. \\ & (V2x = (ap (ap c\_2Einteger\_2Eint\_add (ap (ap c\_2Einteger\_2Eint\_mul (ap c\_2Einteger\_2Eint\_neg V5p)) (ap (ap c\_2Einteger\_2Eint\_div V3b) (ap c\_2Einteger\_2Eint\_of\_num V4d)))) (ap (ap c\_2Einteger\_2Eint\_mul V7t) (ap (ap c\_2Einteger\_2Eint\_div (ap c\_2Einteger\_2Eint\_of\_num V0m)) (ap c\_2Einteger\_2Eint\_of\_num V4d))))))))))))))) \quad (49) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Einteger\_2Eint. (\forall V3q \in ty\_2Einteger\_2Eint. (((ap (ap c\_2Einteger\_2Eint\_add (ap (ap c\_2Einteger\_2Eint\_mul V2p) (ap c\_2Einteger\_2Eint\_of\_num V0m))) (ap (ap c\_2Einteger\_2Eint\_mul V3q) (ap c\_2Einteger\_2Eint\_of\_num V1n))) = (ap c\_2Einteger\_2Eint\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \Rightarrow ((ap (ap c\_2Egcd\_2Egcd V0m) V1n) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))))) \quad (50) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in ty\_2Einteger\_2Eint. ((ap (ap c\_2Einteger\_2Eint\_div V0p) (ap c\_2Einteger\_2Eint\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) = V0p)) \quad (51) \end{aligned}$$

Assume the following.

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& (ap c\_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge (((ap c\_2Enum\_2ESUC \\
& c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO)))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. \\
& (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Enum\_2ESUC V17n)))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& V24n)))))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& V28n)))))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
& V32n)) (ap c\_2Earithmetic\_2ENUMERAL V32n)) \Leftrightarrow False)))) \wedge \\
& ((\forall V33n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D \\
& c\_2Enum\_2E0) V33n)) (ap c\_2Earithmetic\_2ENUMERAL V33n))) \Leftrightarrow True)))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{54}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{55}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
\end{aligned} \tag{58}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p \\
& V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \\
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \\
\end{aligned} \tag{61}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (63)$$

### Theorem 1

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1a \in ty\_2Enum\_2Enum. (\forall V2x \in ty\_2Einteger\_2Eint. (\forall V3b \in ty\_2Einteger\_2Eint. (\forall V4p \in ty\_2Einteger\_2Eint. (\forall V5q \in ty\_2Einteger\_2Eint. (((ap (ap c\_2Einteger\_2Eint\_add (ap (ap c\_2Einteger\_2Eint\_mul V4p) (ap c\_2Einteger\_2Eint\_of\_num V1a))) (ap (ap c\_2Einteger\_2Eint\_mul V5q) (ap c\_2Einteger\_2Eint\_of\_num V0m))) = (ap c\_2Einteger\_2Eint\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2ZERO))) \wedge ((\neg(V0m = c\_2Enum\_2E0)) \wedge (\neg(V1a = c\_2Enum\_2E0)))) \Rightarrow ((p (ap (ap c\_2Einteger\_2Eint\_divides (ap c\_2Einteger\_2Eint\_of\_num V0m)) (ap (ap c\_2Einteger\_2Eint\_add (ap (ap c\_2Einteger\_2Eint\_mul (ap c\_2Einteger\_2Eint\_of\_num V1a)) V2x)) V3b))) \Leftrightarrow (\exists V6t \in ty\_2Einteger\_2Eint. (V2x = (ap (ap c\_2Einteger\_2Eint\_add (ap (ap c\_2Einteger\_2Eint\_mul (ap c\_2Einteger\_2Eint\_neg V4p)) V3b)) (ap (ap c\_2Einteger\_2Eint\_mul V6t) (ap c\_2Einteger\_2Eint\_of\_num V0m)))))))))))))) \end{aligned}$$