thm_2Eint__arith_2Egcdthm2 (TMYYJj14xtoMudYKzpKc7uscLdGBjsuksXm)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (2)$$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Edivides_2Edivides *to be* $\lambda V0a \in ty\_2Enum\_2Enum.\lambda V1b \in ty\_2Enum\_2Enum$

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Egcd_2Eis__gcd *to be* $\lambda V0a \in ty\_2Enum\_2Enum.\lambda V1b \in ty\_2Enum\_2Enum.\lambda V$

Let $c\_2Egcd\_2Egcd : \iota$ be given. Assume the following.

$$c\_2Egcd\_2Egcd \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (3)$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (4)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (5)$$

Definition 10 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 11 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{6}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{7}$$

Definition 12 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{8}$$

Definition 13 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 14 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{9}$$

Let $c\_2Einteger\_2Eint\_\_div : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_div \in ((ty\_2Einteger\_2Eint^{ty\_2Einteger\_2Eint})^{ty\_2Einteger\_2Eint}) \tag{10}$$

Let $c\_2Einteger\_2Eint\_\_mod : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_mod \in ((ty\_2Einteger\_2Eint^{ty\_2Einteger\_2Eint})^{ty\_2Einteger\_2Eint}) \tag{11}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{12}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{13}$$

Definition 15 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ t} \tag{14}$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{15}$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)} \tag{16}$$

Definition 16 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 17 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Intege$

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{17}$$

Definition 18 *We define* c_2Einteger_2Eint__mul *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Intege$

Definition 19 *We define* c_2Einteger_2Eint__divides *to be* $\lambda V0p \in ty\_2Einteger\_2Eint.\lambda V1q \in ty\_2Eintege$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{18}$$

Definition 20 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \tag{19}$$

Definition 21 *We define* c_2Emarker_2EAbbrev *to be* $\lambda V0x \in 2.V0x$.

Definition 22 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 23 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 24 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow ((V0m = \mathsf{c\_2Enum\_2E0}) \lor (V1n = \mathsf{c\_2Enum\_2E0}))))) \tag{20}$$

Assume the following.

$$True \tag{21}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{24}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg (p\ V0t)))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{30}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\forall V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A\_27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \wedge (\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \vee (\exists V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V3x))))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.((\exists V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (p\ V1Q))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\exists V2x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{42}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \tag{43}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A))))) \tag{44}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{45}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{46}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in A\_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b^{A\_27a}).(\forall V4x \in A\_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \tag{48}$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1b \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Egcd\_2Eis\_\_gcd}\ V0a)\ V1b)\ (ap\ (ap\ c\_2Egcd\_2Egcd\ V0a)\ V1b))))) \tag{49}$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1b \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Egcd\_2Egcd\ V0a)\ V1b) = (ap\ (ap\ c\_2Egcd\_2Egcd\ V1b)\ V0a)))) \tag{50}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((\neg(V0n = \mathsf{c\_2Enum\_2E0}))\wedge(\neg(V1m = \mathsf{c\_2Enum\_2E0})))\Rightarrow(\exists V2p \in ty\_2Enum\_2Enum.(\exists V3q \in ty\_2Enum\_2Enum.((V0n = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V2p)\ (ap\ (ap\ c\_2Egcd\_2Egcd\ V0n)\ V1m)))\wedge((V1m = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V3q)\ (ap\ (ap\ c\_2Egcd\_2Egcd\ V0n)\ V1m)))\wedge((ap\ (ap\ c\_2Egcd\_2Egcd\ V2p)\ V3q) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))))))) \tag{51}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2z \in ty\_2Einteger\_2Eint.((V0x = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1y)\ V2z))\Leftrightarrow((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1y)) = V2z))))) \tag{52}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Edivides\_2Edivides}\ V0n)\ V1m))))) \tag{53}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1q \in ty\_2Einteger\_2Eint.(\forall V2r \in ty\_2Einteger\_2Eint.((\neg(V1q = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0p)\ V1q))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2r)\ V1q)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ V0p)\ V2r))))))) \tag{54}$$

Assume the following.

$$(\forall V0p \in ty\_2Enum\_2Enum.(\forall V1q \in ty\_2Enum\_2Enum.(\forall V2r \in ty\_2Einteger\_2Eint.(((ap\ (ap\ c\_2Egcd\_2Egcd\ V0p)\ V1q) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0p))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1q))\ V2r)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0p))\ V2r))))))) \tag{55}$$

Assume the following.

$$(\forall V0y \in ty\_2Einteger\_2Eint.(\forall V1x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1x)\ V0y) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0y)\ V1x)))) \tag{56}$$

Assume the following.

$$(\forall V0y \in ty\_2Einteger\_2Eint.(\forall V1x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V1x)\ V0y) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0y)\ V1x)))) \tag{57}$$

Assume the following.

$$(\forall V0z \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V2x)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1y)\ V0z)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V2x)\ V1y))\ V0z))))) \tag{58}$$

Assume the following.

$$(\forall V0z \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2x)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V1y)\ V0z)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2x)\ V1y))\ V0z))))) \tag{59}$$

Assume the following.

$$(\forall V0z \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2x)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1y)\ V0z)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2x)\ V1y))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2x)\ V0z)))))) \tag{60}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V0x) = V0x)) \tag{61}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) = V0x)) \tag{62}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2z \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ V1y))\ V2z) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V1y)\ V2z)))))) \tag{63}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x))\ V1y)))) \tag{64}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2z \in ty\_2Einteger\_2Eint.(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V2z) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V1y)\ V2z))\Leftrightarrow((V2z = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\vee(V0x = V1y)))))) \tag{65}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n))\Leftrightarrow(V0m = V1n)))) \tag{66}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))))) \tag{67}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.((ap\ (ap\ c\_2Einteger\_2Eint\_\_div\ V0p)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) = V0p)) \tag{68}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.((\neg(V0p = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Rightarrow((ap\ (ap\ c\_2Einteger\_2Eint\_\_div\ V0p)\ V0p) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \tag{69}$$

Assume the following.

$$(\forall V0i \in ty\_2Einteger\_2Eint.((\neg(V0i = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Rightarrow((ap\ (ap\ c\_2Einteger\_2Eint\_\_mod\ V0i)\ V0i) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))) \tag{70}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1q \in ty\_2Einteger\_2Eint.(\forall V2k \in ty\_2Einteger\_2Eint.(((\neg(V1q = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\wedge((ap\ (ap\ c\_2Einteger\_2Eint\_\_mod\ V0p)\ V1q) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Rightarrow((ap\ (ap\ c\_2Einteger\_2Eint\_\_div\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2k)\ V0p))\ V1q) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2k)\ (ap\ (ap\ c\_2Einteger\_2Eint\_\_div\ V0p)\ V1q)))))))) \tag{71}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1q \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ V0p)\ V1q))\Leftrightarrow((((ap\ (ap\ c\_2Einteger\_2Eint\_\_mod\ V1q)\ V0p) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\wedge(\neg(V0p = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))\vee((V0p = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\wedge(V1q = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))))) \tag{72}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ V0x)\ V0x))) \tag{73}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2z \in ty\_2Einteger\_2Eint.(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ V0x)\ V1y))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ V1y)\ V2z)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ V0x)\ V2z)))))) \tag{74}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1q \in ty\_2Einteger\_2Eint.(\forall V2r \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ V0p)\ V1q))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ V0p)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V1q)\ V2r))))))) \tag{75}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1q \in ty\_2Einteger\_2Eint.(\forall V2r \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ V0p)\ V1q))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ V0p)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2r)\ V1q))))))) \tag{76}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1q \in ty\_2Einteger\_2Eint.(\forall V2r \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ V0p)\ V1q))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ V0p)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1q)\ V2r)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ V0p)\ V2r)))))))\tag{77}$$

Assume the following.

$$((\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)))))\wedge((\forall V2x \in ty\_2Einteger\_2Eint.(\forall V3y \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V2x))\ V3y) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2x)\ V3y)))))\wedge((\forall V4x \in ty\_2Einteger\_2Eint.(\forall V5y \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V4x)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V5y)) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V4x)\ V5y)))))\wedge(\forall V6x \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V6x)) = V6x)))))\tag{78}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\tag{79}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False)))\tag{80}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow(((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False)))))\tag{81}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False)))))\tag{82}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False)))\tag{83}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))))\tag{84}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{85}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{86}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{87}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{88}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{89}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{90}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{91}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{92}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{93}$$

Theorem 1

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1a \in ty\_2Enum\_2Enum.(\forall V2x \in ty\_2Einteger\_2Eint.(\forall V3b \in ty\_2Einteger\_2Eint.(\forall V4d \in ty\_2Enum\_2Enum.(\forall V5p \in ty\_2Einteger\_2Eint.(\forall V6q \in ty\_2Einteger\_2Eint.(((V4d = (ap\ (ap\ c\_2Egcd\_2Egcd\ V1a)\ V0m))\wedge(((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V4d) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V5p)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1a)))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V6q)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m))))\wedge((\neg(V4d = \mathsf{c\_2Enum\_2E0}))\wedge((\neg(V0m = \mathsf{c\_2Enum\_2E0}))\wedge(\neg(V1a = \mathsf{c\_2Enum\_2E0}))))))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1a))\ V2x))\ V3b)))\Leftrightarrow((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_divides}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V4d))\ V3b))\wedge(\exists V7t \in ty\_2Einteger\_2Eint.(V2x = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V5p))\ (ap\ (ap\ c\_2Einteger\_2Eint\_\_div\ V3b)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V4d))))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V7t)\ (ap\ (ap\ c\_2Einteger\_2Eint\_\_div\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V4d))))))))))))))))$$