

thm_2Eint__arith_2Ejustify__divides2 (TMZUM4SCZyMroCA1ATe92k3w6Yyjga8Kt4z)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Eenum_2E_enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2E_enum \tag{1}$$

Let $ty_2Epair_2E_eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2E_eprod\ A0\ A1) \tag{2}$$

Let $ty_2Einteger_2E_eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2E_eint \tag{3}$$

Let $c_2Einteger_2E_eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2E_eint_REP_CLASS \in ((2^{(ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum)})^{ty_2Einteger_2E_eint}) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p)$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define $c_2Einteger_2E_eint_REP$ to be $\lambda V0a \in ty_2Einteger_2E_eint.(ap (c_2Emin_2E_40 (ty$

Let $c_2Einteger_2E_etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2E_etint_add \in (((ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum)^{ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum})^{ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum}) \tag{5}$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (7)$$

Definition 6 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 7 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (8)$$

Definition 8 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (9)$$

Definition 9 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_2Einteger_2Eint_divides$ to be $\lambda V0p \in ty_2Einteger_2Eint.\lambda V1q \in ty_2Einteger$

Definition 12 We define $c_2Ebool_2E_2F$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 15 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).((\neg(\exists V1x \in A.27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p\ (ap\ V0P\ V2x)))))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B))))))) \quad (26)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint.(\forall V2z \in ty_2Einteger_2Eint.((V0x = (ap\ (ap\ c_2Einteger_2Eint_add\ V1y)\ V2z)) \Leftrightarrow ((ap\ (ap\ c_2Einteger_2Eint_add\ V0x)\ (ap\ c_2Einteger_2Eint_neg\ V1y)) = V2z)))))) \quad (27)$$

Assume the following.

$$(\forall V0y \in ty_2Einteger_2Eint.(\forall V1x \in ty_2Einteger_2Eint.((ap\ (ap\ c_2Einteger_2Eint_mul\ V1x)\ V0y) = (ap\ (ap\ c_2Einteger_2Eint_mul\ V0y)\ V1x)))) \quad (28)$$

Assume the following.

$$(\forall V0z \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint.(\forall V2x \in ty_2Einteger_2Eint.((ap\ (ap\ c_2Einteger_2Eint_mul\ V2x)\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V1y)\ V0z)) = (ap\ (ap\ c_2Einteger_2Eint_mul\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V2x)\ V1y))\ V0z)))))) \quad (29)$$

Assume the following.

$$(\forall V0z \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint.(\forall V2x \in ty_2Einteger_2Eint.((ap\ (ap\ c_2Einteger_2Eint_mul\ V2x)\ (ap\ (ap\ c_2Einteger_2Eint_add\ V1y)\ V0z)) = (ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V2x)\ V1y))\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V2x)\ V0z)))))) \quad (30)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap\ c_2Einteger_2Eint_neg\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x) \\
& V1y)) = (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x)\ (ap\ c_2Einteger_2Eint_neg \\
& V1y))))))
\end{aligned} \tag{31}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{32}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{35}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(\\
& p\ V2r)) \vee (\neg(p\ V1q)))))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\
& ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge (\\
& \neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee \\
& (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))
\end{aligned} \tag{40}$$

Theorem 1

$$\begin{aligned} & (\forall V0n \in ty_2Einteger_2Eint. (\forall V1c \in ty_2Einteger_2Eint. \\ & (\forall V2x \in ty_2Einteger_2Eint. (\forall V3y \in ty_2Einteger_2Eint. \\ & ((p (ap (ap c_2Einteger_2Eint_divides (ap (ap c_2Einteger_2Eint_mul \\ & V0n) V2x)) (ap (ap c_2Einteger_2Eint_add (ap (ap c_2Einteger_2Eint_mul \\ & V0n) V3y)) V1c))) \Leftrightarrow ((p (ap (ap c_2Einteger_2Eint_divides (ap (\\ & ap c_2Einteger_2Eint_mul V0n) V2x)) (ap (ap c_2Einteger_2Eint_add \\ & (ap (ap c_2Einteger_2Eint_mul V0n) V3y)) V1c))) \wedge (p (ap (ap c_2Einteger_2Eint_divides \\ & V0n) V1c)))))))))) \end{aligned}$$