

thm\_2Eint\_bitwise\_2Eint\_bit\_shift\_left  
 (TMGZ9cyTT5t6RvmDhsvyo8Vcru3bByZF1bk)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x))$

**Definition 3** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 4** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a}) A\_27a) A\_27a) A\_27a)$

**Definition 5** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0f \in (A\_27b^{A\_27a}). (\lambda V1x \in A\_27a. f x))$

**Definition 6** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x. (V0x = V1x)))$

**Definition 7** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}) A\_27a) A\_27a) P))$

**Definition 8** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (3)$$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x)) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Einteger\_2EEnum$  to be  $\lambda V0i \in ty\_2Einteger\_2Eint. (ap (c\_2Emin\_2E\_40 ty\_2Eint) i)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (4)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (5)$$

**Definition 11** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 12** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (6)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (7)$$

**Definition 13** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 14** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n)\ V0)$

**Definition 15** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty\_2Elist\_2Elist\ A0) \quad (11)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (12)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (13)$$

**Definition 16** We define  $c_{\text{2Emin\_3D\_3D\_3E}}$  to be  $\lambda P \in 2.\lambda Q \in 2.\text{inj\_o} (p \Rightarrow p Q)$  of type  $\iota$ .

**Definition 17** We define  $c_{\text{Ebool\_2E\_2F\_5C}}$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_{\text{Ebool\_2E\_21}}) 2)) (\lambda V2t \in$

**Definition 19** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E))$

**Definition 20** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\;V0P\;(ap\;(c\_2Emin\_2E\_40$

**Definition 21** We define  $c\_2Erelation\_2EWF$  to be  $\lambda A.27a : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).(ap\ (c\_2Ebool\_2E\_21$

Let  $c\_Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$\forall A \exists a \text{ nonempty } A \exists a \Rightarrow c \in E \text{ bool } \exists EABB A \exists a$

— 22 — W. 1. 6. — 25-141. — **SUPERSTITION** (1) 34. 27. — 34. 27.

**DEFINITION 2.1** We define  $\mathcal{C}$ -acceleration to be  $X\mathcal{A}=T^*a : t \cdot X\mathcal{A}=T^*a : t \cdot X$  for  $t \in ((2^{-n}, 1])$ .

**Definition 25** We define  $\mathcal{C}_2$ -relation  $\mathcal{R}$  the run to be  $\lambda A \cdot \mathcal{R}d : t \cdot \lambda x \cdot \mathcal{R}b : t \cdot \lambda V \circ R \in ((\mathcal{C}_2 \text{-run}) \rightarrow \lambda V \circ T)$

**Definition 26** We define  $\text{c\_2ERelation\_2EWREC}$  to be  $\lambda A_27a : i. \lambda A_27b : i. \lambda V0R \in ((2^{A_27a})^i). \lambda V1M$

**Definition 27** We define `c_2EInt_bitwise_2Ebits_of_num` to be  $(ap \ (ap \ (c_2Erelation_2EWREC \ ty\_2Enum))$

Let  $ty\_ZEpair\_ZEprod : t \Rightarrow t \Rightarrow t$  be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty}(\text{ty\_2Epair\_2Eprod } A0\ A1) \quad (15)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{\text{--}27a}. \text{nonempty } A_{\text{--}27a} \Rightarrow \forall A_{\text{--}27b}. \text{nonempty } A_{\text{--}27b} \Rightarrow c_{\text{--}2Epair\text{--}2EABS\text{--}prod} \\ A_{\text{--}27a} \ A_{\text{--}27b} \in ((ty_{\text{--}2Epair\text{--}2Eprod} \ A_{\text{--}27a} \ A_{\text{--}27b})^{((2^{A_{\text{--}27b}})^{A_{\text{--}27a}})}) \quad (16)$$

**Definition 28** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

**Definition 29** We define  $c\_2Earthmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earthmetic\ n\ 0)\ V)$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_2Eprod\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\_ty\_2Enum\_2Enum\_ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eprod\_REP\_CLASS})$$

**Definition 30** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E\_40 (t_0))$   
Let  $c\_2Einteger\_2Etint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_neg \in ((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (18)$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (19)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)}} \quad (20)$$

**Definition 31** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)$

**Definition 32** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap c\_2Einteger\_2Eint\_neg T1)$

Let  $c\_2Einteger\_2Etint\_add : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_add \in (((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (21)$$

**Definition 33** We define  $c\_2Einteger\_2Eint\_add$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint.ap (c\_2Einteger\_2Etint\_add T1) T2$

**Definition 34** We define  $c\_2Einteger\_2Eint\_sub$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2Eint.ap (c\_2Einteger\_2Eint\_add (c\_2Einteger\_2Eint\_neg V1y)) V0x$

**Definition 35** We define  $c\_2Eint\_bitwise\_2Eint\_not$  to be  $\lambda V0i \in ty\_2Einteger\_2Eint.(ap (ap c\_2Einteger\_2Eint\_not i))$

Let  $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Elist\_2EMAP A\_27a A\_27b \in (((ty\_2Elist\_2Elist A\_27b)^{(ty\_2Elist\_2Elist A\_27a)})^{(A\_27b^{A\_27a})}) \quad (22)$$

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (23)$$

**Definition 36** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint.ap (c\_2Einteger\_2Etint\_lt T1) T2$

**Definition 37** We define  $c\_2Eint\_bitwise\_2Ebits\_of\_int$  to be  $\lambda V0i \in ty\_2Einteger\_2Eint.(ap (ap (c\_2Eint\_bitwise\_2Ebits\_of\_int i)))$

Let  $c\_2Elist\_2EGENLIST : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EGENLIST A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{ty\_2Enum\_2Enum})^{(A\_27a^{ty\_2Enum\_2Enum})}) \quad (24)$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{\_27a}. \text{nonempty } A_{\_27a} \Rightarrow c_{\_2}Elist\_2EAPPEND\ A_{\_27a} \in (((ty\_2Elist\_2Elist\ A_{\_27a})^{(ty\_2Elist\_2Elist\ A_{\_27a})})^{(ty\_2Elist\_2Elist\ A_{\_27a})}) \quad (25)$$

Let  $c\_2Eint\_bitwise\_2Eint\_of\_bits : \iota$  be given. Assume the following.

$$c\_2Eint\_bitwise\_2Eint\_of\_bits \in (ty\_2Einteger\_2Eint(ty\_2Epair\_2Eprod\ (ty\_2Elist\_2Elist\ 2)\ 2)) \quad (26)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a \ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod \ A\_27a \ A\_27b)}) \quad (27)$$

Let  $c_{\text{2Pair-2EFST}} : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{\_27a}.nonempty\ A_{\_27a} \Rightarrow \forall A_{\_27b}.nonempty\ A_{\_27b} \Rightarrow c_{\_2Epair\_2EFST}(A_{\_27a}, A_{\_27b}) \in (A_{\_27a}^{(ty\_2Epair\_2Eprod\ A_{\_27a}\ A_{\_27b})}) \quad (28)$$

**Definition 38** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in ((A.27c^A.27b)^{\iota} \rightarrow \iota.\lambda V0f)$

**Definition 39** We define  $c\_2Eint\_bitwise\_2Eint\_shift\_left$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1i \in ty\_2Enum\_2Enum.$

Let  $c_2 \in \text{arithmetic\_EXP} : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (29)$$

**Definition 40** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

Let  $c_2Earithmetic_2E_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (30)$$

Let  $c_2Earithmetic_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (31)$$

**Definition 41** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 42** We define  $c\_2EBit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2Enum\_2Enum. \lambda V2m \in ty\_2Enum\_2Enum. \lambda V3n \in ty\_2Enum\_2Enum. \lambda V4o \in ty\_2Enum\_2Enum. \lambda V5p \in ty\_2Enum\_2Enum. \lambda V6q \in ty\_2Enum\_2Enum. \lambda V7r \in ty\_2Enum\_2Enum. \lambda V8s \in ty\_2Enum\_2Enum. \lambda V9t \in ty\_2Enum\_2Enum. \lambda V10u \in ty\_2Enum\_2Enum. \lambda V11v \in ty\_2Enum\_2Enum. \lambda V12w \in ty\_2Enum\_2Enum. \lambda V13x \in ty\_2Enum\_2Enum. \lambda V14y \in ty\_2Enum\_2Enum. \lambda V15z \in ty\_2Enum\_2Enum.$

**Definition 43** We define  $c\_2EBit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap$

**Definition 44** We define  $c\_2Eint\_bitwise\_2Eint\_bit$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1i \in ty\_2Einteger.$

Let  $c_2$  be given. Assume the following.

*c* ?E<sub>arithmeti</sub>c ?E<sub>EVEN</sub> ∈ (2<sup>ty-2Enum-2Enum</sup>)

**Definition 45** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 46** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 47** We define  $c_{\text{C\_Ebool\_2E\_5C\_2F}}$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_{\text{C\_Ebool\_2E\_21}})2))(\lambda V2t \in$

**Definition 48** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 49** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (ap\ (c\_2Ebool\_2E$

Let  $c_2$  be given. Assume the following.

$c : 2Earithmic \cdot 2E \cdot 2A \in ((tu : 2Enum \cdot 2Enum)^{ty\_2Enum\_2Enum})$

(33)

Let  $\langle 2EL \rangle + \langle 2EEL \rangle = 1$ , i.e.,  $A = 1$ , the following

(34)

$$\forall A \in \mathcal{A}, \text{nonempty } A \subseteq \{a \in \mathcal{A} : \text{exists } z \in \mathcal{Z} \text{ such that } H(A, z) \in \{y \in \mathcal{Y} : \text{exists } z \in \mathcal{Z} \text{ such that } H(A, z) = y\}\} \quad (35)$$

**Definition 31** We define  $c_2$  Earthmetics $\leq_{\text{SC-SD}}$  to be  $\lambda V \; \theta m \in ig\_Z \text{Earthmetics} \; \lambda V \; m \in ig\_Z$

Assume the following.

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (38)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & \quad \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
 & \quad (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
 & \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p)))))) \\
 \end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & \quad (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & \quad (ap c\_2Enum\_2ESUC V0m)) V1n)))))) \\
 \end{aligned} \tag{40}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 c\_2Enum\_2E0) V0n))) \tag{41}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & \quad p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) (ap (ap c\_2Earithmetic\_2E\_2B \\
 & \quad V0m) V1n)))))) \\
 \end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & \quad (\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & \quad V1n) V0m)))))) \\
 \end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
 & \quad (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
 & \quad (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
 & \quad (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
 & \quad (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
 & \quad (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge \\
 & \quad ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
 & \quad (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
 & \quad V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
 & \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
 & \quad V0m) V1n)))))))))) \\
 \end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & \quad \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & \quad V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
 & \quad ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p)))))) \\
 \end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum. (\forall V1c \in ty\_2Enum\_2Enum. (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2B V0a) V1c)) V1c) = V0a))) \quad (46)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \Rightarrow (\exists V2p \in ty\_2Enum\_2Enum. (V1n = (ap (ap c_2Earithmetic_2E_2B V0m) V2p)))))) \quad (47)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (48)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))) \quad (49)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V1n)) V0m)))))) \quad (50)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0n))) \quad (51)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}). (\forall V1a \in ty\_2Enum\_2Enum. (\forall V2b \in ty\_2Enum\_2Enum. ((p (ap V0P (ap (ap c_2Earithmetic_2E_2D V1a) V2b))) \Leftrightarrow (\forall V3d \in ty\_2Enum\_2Enum. (((V2b = (ap (ap c_2Earithmetic_2E_2B V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0))) \wedge ((V1a = (ap (ap c_2Earithmetic_2E_2B V2b) V3d)) \Rightarrow (p (ap V0P V3d)))))))))) \quad (52)$$

Assume the following.

$$True \quad (53)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (55)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \\ & \forall V0f \in (A\_27b^{A\_27a}). (\forall V1x \in A\_27a. ((ap (ap (c\_2Ebool\_2ELT \\ & A\_27a A\_27b) V0f) V1x) = (ap V0f V1x))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ & A\_27a. (p V0t) \Leftrightarrow (p V0t))) \end{aligned} \quad (58)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge \\ ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow \\ & (p V0t)) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (62)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \quad (63)$$

Assume the following.

$$\forall A_{\text{27}}a.\text{nonempty } A_{\text{27}}a \Rightarrow (\forall V0x \in A_{\text{27}}a. ((V0x = V0x) \Leftrightarrow \text{True})) \quad (64)$$

Assume the following.

$$\forall A_{\text{27}}a.\text{nonempty } A_{\text{27}}a \Rightarrow (\forall V0x \in A_{\text{27}}a. (\forall V1y \in A_{\text{27}}a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (65)$$

Assume the following.

$$(\forall V0t \in 2. (((\text{True} \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow \text{True}) \Leftrightarrow (p V0t)) \wedge (((\text{False} \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(p V0t))))))) \quad (66)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow (\neg(p V0A) \vee (\neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow (\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (69)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow \text{False}) \Leftrightarrow ((p V0t) \Leftrightarrow \text{False}))) \quad (70)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (71)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2))))))) \quad (72)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{\text{27}} \in 2. (\forall V2y \in 2. (\forall V3y_{\text{27}} \in 2. (((p V0x) \Leftrightarrow (p V1x_{\text{27}})) \wedge ((p V1x_{\text{27}}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{\text{27}})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{\text{27}}) \Rightarrow (p V3y_{\text{27}}))))))) \quad (73)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a.(((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\ & V5y\_27))))))) \end{aligned} \quad (74)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow ((\forall V0t1 \in A\_27a.(\forall V1t2 \in \\ & A\_27a.((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap \\ & (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V2t1)\ V3t2) = V3t2)))))) \end{aligned} \quad (75)$$

Assume the following.

$$(\forall V0P \in (2^2).((\forall V1b \in 2.(p\ (ap\ V0P\ V1b))) \Leftrightarrow ((p\ (ap\ V0P\ c\_2Ebool\_2ET)) \wedge (p\ (ap\ V0P\ c\_2Ebool\_2EF)))))) \quad (76)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ & \forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (ap\ (c\_2Ecombin\_2EK \\ & A\_27a\ A\_27b)\ V0x)\ V1y) = V0x))) \end{aligned} \quad (77)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (c\_2Ecombin\_2EI \\ & A\_27a)\ V0x) = V0x)) \quad (78)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1b \in (ty\_2Epair\_2Eprod \\ & (ty\_2Elist\_2Elist\ 2)\ 2).((p\ (ap\ (ap\ c\_2Eint\_bitwise\_2Eint\_bit \\ & V0n)\ (ap\ c\_2Eint\_bitwise\_2Eint\_of\_bits\ V1b))) \Leftrightarrow (p\ (ap\ (ap \\ & ap\ (c\_2Ebool\_2ECOND\ 2)\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ (ap\ (c\_2Elist\_2ELENGTH \\ & 2)\ (ap\ (c\_2Epair\_2EFST\ (ty\_2Elist\_2Elist\ 2)\ 2)\ V1b))))\ (ap\ ( \\ & ap\ (c\_2Elist\_2EEEL\ 2)\ V0n)\ (ap\ (c\_2Epair\_2EFST\ (ty\_2Elist\_2Elist \\ & 2)\ 2)\ V1b)))\ (ap\ (c\_2Epair\_2ESND\ (ty\_2Elist\_2Elist\ 2)\ 2)\ V1b))))))) \end{aligned} \quad (79)$$

Assume the following.

$$(\forall V0i \in ty\_2Einteger\_2Eint.((ap\ c\_2Eint\_bitwise\_2Eint\_of\_bits \\ & (ap\ c\_2Eint\_bitwise\_2Ebits\_of\_int\ V0i)) = V0i)) \quad (80)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0l1 \in (ty\_2Elist\_2Elist \\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist A\_27a).((ap (c\_2Elist\_2ELENGTH \\ A\_27a) (ap (ap (c\_2Elist\_2EAPPEND A\_27a) V0l1) V1l2)) = (ap (ap c\_2Earithmetic\_2E\_2B \\ (ap (c\_2Elist\_2ELENGTH A\_27a) V0l1)) (ap (c\_2Elist\_2ELENGTH A\_27a) \\ V1l2))))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0f \in (A\_27a^{ty\_2Enum\_2Enum}). \\ (\forall V1n \in ty\_2Enum\_2Enum.((ap (c\_2Elist\_2ELENGTH A\_27a) & (ap (ap (c\_2Elist\_2EGENLIST A\_27a) V0f) V1n)) = V1n))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0f \in (A\_27a^{ty\_2Enum\_2Enum}). \\ (\forall V1n \in ty\_2Enum\_2Enum.(\forall V2x \in ty\_2Enum\_2Enum.(& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V2x) V1n)) \Rightarrow ((ap (ap (c\_2Elist\_2EE \\ A\_27a) V2x) (ap (ap (c\_2Elist\_2EGENLIST A\_27a) V0f) V1n)) = (ap V0f \\ V2x))))))) \end{aligned} \quad (83)$$

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A V6n) V7m))))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D V10n) V11m))))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP V14n) c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))) \wedge ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2EEEXP V15n) V16m))))))) \wedge (((ap c\_2Enum\_2ESUC c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enum\_2ESUC V17n))))))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Eprim\_rec\_2EPRE V18n))))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum.((\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m))))))) \wedge ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C V23n) c\_2Enum\_2E0)) \Leftrightarrow False))) \wedge ((\forall V24n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) V24n))))))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.((\forall V26m \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C V25n) c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C V25n) V26m))))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E c\_2Enum\_2E0) V27n)) \Leftrightarrow False))) \wedge ((\forall V28n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E V28n) c\_2Enum\_2E0) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3E c\_2Enum\_2E0) V28n))))))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.((\forall V30m \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C V29n) c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C V29n) V30m))))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C c\_2Enum\_2E0) V31n)) \Leftrightarrow True))) \wedge ((\forall V32n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C c\_2Enum\_2E0) V32n)) \Leftrightarrow False))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C c\_2Enum\_2E0) V32n))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C c\_2Enum\_2E0) V32n)) \Leftrightarrow False)))$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \\
& \forall V0x \in (ty\_2Epair\_2Eprod A\_27a A\_27b). (\exists V1q \in A\_27a. \\
& (\exists V2r \in A\_27b. (V0x = (ap (ap (c_2Epair\_2E_2C A\_27a A\_27b) \\
& V1q) V2r)))) \\
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \\
& \forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap (c_2Epair\_2EFST A\_27a \\
& A\_27b) (ap (ap (c_2Epair\_2E_2C A\_27a A\_27b) V0x) V1y)) = V0x)) \\
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \\
& \forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap (c_2Epair\_2ESND A\_27a \\
& A\_27b) (ap (ap (c_2Epair\_2E_2C A\_27a A\_27b) V0x) V1y)) = V1y)) \\
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \forall A\_27c. \\
& \quad \text{nonempty } A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}). (\forall V1x \in \\
& \quad A\_27a. (\forall V2y \in A\_27b. ((ap (ap (c_2Epair\_2EUNCURRY A\_27a \\
& A\_27b A\_27c) V0f) (ap (ap (c_2Epair\_2E_2C A\_27a A\_27b) V1x) V2y)) = \\
& \quad (ap (ap V0f V1x) V2y)))))) \\
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0n \in ty\_2Enum\_2Enum. \\ & \forall V1l1 \in (ty\_2Elist\_2Elist A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist \\ & A\_27a).((p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) (ap (c\_2Elist\_2ELENGTH \\ & A\_27a) V1l1))) \Rightarrow ((ap (ap (c\_2Elist\_2EEEL A\_27a) V0n) (ap (ap (c\_2Elist\_2EAPPEND \\ & A\_27a) V1l1) V2l2)) = (ap (ap (c\_2Elist\_2EEEL A\_27a) V0n) V1l1)))))) \\ & (90) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0l1 \in (ty\_2Elist\_2Elist \\ & A\_27a).(\forall V1n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & (ap (c\_2Elist\_2ELENGTH A\_27a) V0l1)) V1n)) \Rightarrow (\forall V2l2 \in (ty\_2Elist\_2Elist \\ & A\_27a).((ap (ap (c\_2Elist\_2EEEL A\_27a) V1n) (ap (ap (c\_2Elist\_2EAPPEND \\ & A\_27a) V0l1) V2l2)) = (ap (ap (c\_2Elist\_2EEEL A\_27a) (ap (ap c\_2Earithmetic\_2E\_2D \\ & V1n) (ap (c\_2Elist\_2ELENGTH A\_27a) V0l1))) V2l2)))))) \\ & (91) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (92)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (93)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (94)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (95)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (96)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee ((\neg(p V1q)) \vee ((\neg(p V0p))))))) \wedge ((p V2r) \vee \\ ((\neg(p V1q)) \vee ((\neg(p V0p))))))))))) \quad (97) \end{aligned}$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee ((\neg(p V2r))))))) \wedge (((p V1q) \vee \\ ((\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V0p)))))))))) \quad (98) \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
 & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
 & \tag{99}
 \end{aligned}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
 (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \tag{100}$$

### Theorem 1

$$\begin{aligned}
 & (\forall V0b \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & \forall V2i \in ty\_2Einteger\_2Eint. ((p (ap (ap c\_2Eint\_bitwise\_2Eint\_bit \\
 & V0b) (ap (ap c\_2Eint\_bitwise\_2Eint\_shift\_left V1n) V2i))) \Leftrightarrow \\
 & ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0b)) \wedge (p (ap (ap c\_2Eint\_bitwise\_2Eint\_bit \\
 & (ap (ap c\_2Earithmetic\_2E\_2D V0b) V1n)) V2i)))))))
 \end{aligned}$$