

thm_2IntegerRing_2Eint__calculate (TMbpTyFpjALkssTucYiZpvJr5tz413s6pnn)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \tag{3}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{5}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 14 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (7)$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (8)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (9)$$

Definition 15 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E_40\ (ty_2Einteger_2Eint_REP_CLASS$

Let $c_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (10)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (11)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}} \quad (12)$$

Definition 16 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 17 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (14)$$

Definition 18 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (15)$$

Definition 19 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (16)$$

Definition 20 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (17)$$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t))) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Einteger_2Eint. (\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2m \in ty_2Enum_2Enum. (((ap (ap c_2Einteger_2Eint_add \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0p) = V0p) \wedge (((\\
& ap (ap c_2Einteger_2Eint_add V0p) (ap c_2Einteger_2Eint_of_num \\
c_2Enum_2E0)) = V0p) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\
V1n)) (ap c_2Einteger_2Eint_of_num V2m)) = (ap c_2Einteger_2Eint_of_num \\
(ap (ap c_2Earithmetic_2E_2B V1n) V2m))) \wedge (((ap (ap c_2Einteger_2Eint_add \\
(ap c_2Einteger_2Eint_of_num V1n)) (ap c_2Einteger_2Eint_neg \\
(ap c_2Einteger_2Eint_of_num V2m))) = (ap (ap (ap (c_2Ebool_2ECOND \\
ty_2Einteger_2Eint) (ap (ap c_2Earithmetic_2E_3C_3D V2m) V1n)) \\
(ap c_2Einteger_2Eint_of_num (ap (ap c_2Earithmetic_2E_2D \\
V1n) V2m))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
(ap (ap c_2Earithmetic_2E_2D V2m) V1n)))))) \wedge (((ap (ap c_2Einteger_2Eint_add \\
(ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V1n))) \\
(ap c_2Einteger_2Eint_of_num V2m)) = (ap (ap (ap (c_2Ebool_2ECOND \\
ty_2Einteger_2Eint) (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2m)) \\
(ap c_2Einteger_2Eint_of_num (ap (ap c_2Earithmetic_2E_2D \\
V2m) V1n))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
(ap (ap c_2Earithmetic_2E_2D V1n) V2m)))))) \wedge (((ap (ap c_2Einteger_2Eint_add \\
(ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V1n))) \\
(ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V2m))) = \\
(ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num (\\
ap (ap c_2Earithmetic_2E_2B V1n) V2m))))))))))))) \\
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_of_num \\
V0m)) (ap c_2Einteger_2Eint_of_num V1n)) = (ap c_2Einteger_2Eint_of_num \\
(ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))) \wedge ((\forall V2x \in ty_2Einteger_2Eint. \\
& (\forall V3y \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
(ap c_2Einteger_2Eint_neg V2x)) V3y) = (ap c_2Einteger_2Eint_neg \\
(ap (ap c_2Einteger_2Eint_mul V2x) V3y)))))) \wedge ((\forall V4x \in ty_2Einteger_2Eint. \\
& (\forall V5y \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
V4x) (ap c_2Einteger_2Eint_neg V5y)) = (ap c_2Einteger_2Eint_neg \\
(ap (ap c_2Einteger_2Eint_mul V4x) V5y)))))) \wedge ((\forall V6x \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_neg V6x)) = \\
V6x)))))) \\
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& (((ap\ c_2Einteger_2Eint_of_num\ V0m) = (ap\ c_2Einteger_2Eint_of_num \\
& \quad V1n)) \Leftrightarrow (V0m = V1n)))) \wedge ((\forall V2x \in ty_2Einteger_2Eint. (\forall V3y \in \\
& ty_2Einteger_2Eint. (((ap\ c_2Einteger_2Eint_neg\ V2x) = (ap\ c_2Einteger_2Eint_neg \\
& \quad V3y)) \Leftrightarrow (V2x = V3y)))) \wedge (\forall V4n \in ty_2Enum_2Enum. (\forall V5m \in \\
& ty_2Enum_2Enum. (((ap\ c_2Einteger_2Eint_of_num\ V4n) = (ap \\
& \quad c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num\ V5m)) \Leftrightarrow \\
& ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))) \wedge (((ap\ c_2Einteger_2Eint_neg \\
& \quad (ap\ c_2Einteger_2Eint_of_num\ V4n)) = (ap\ c_2Einteger_2Eint_of_num \\
& \quad V5m)) \Leftrightarrow ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))))))))) \\
& \hspace{15em} (23)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& \quad \forall V2x \in ty_2Einteger_2Eint. (((ap (ap c_2Einteger_2Eint_add \\
& \quad (ap c_2Einteger_2Eint_of_num V0n)) (ap c_2Einteger_2Eint_of_num \\
& \quad V1m)) = (ap c_2Einteger_2Eint_of_num (ap (ap c_2Earithmetic_2E_2B \\
& \quad V0n) V1m)))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& \quad (ap c_2Einteger_2Eint_of_num V0n))) (ap c_2Einteger_2Eint_of_num \\
& \quad V1m)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap (ap \\
& \quad c_2Earithmetic_2E_3C_3D V0n) V1m)) (ap c_2Einteger_2Eint_of_num \\
& \quad (ap (ap c_2Earithmetic_2E_2D V1m) V0n))) (ap c_2Einteger_2Eint_neg \\
& \quad (ap c_2Einteger_2Eint_of_num (ap (ap c_2Earithmetic_2E_2D \\
& \quad V0n) V1m)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\
& \quad V0n)) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& \quad V1m))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap (\\
& \quad ap c_2Earithmetic_2E_3C_3D V1m) V0n)) (ap c_2Einteger_2Eint_of_num \\
& \quad (ap (ap c_2Earithmetic_2E_2D V0n) V1m))) (ap c_2Einteger_2Eint_neg \\
& \quad (ap c_2Einteger_2Eint_of_num (ap (ap c_2Earithmetic_2E_2D \\
& \quad V1m) V0n)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& \quad (ap c_2Einteger_2Eint_of_num V0n))) (ap c_2Einteger_2Eint_neg \\
& \quad (ap c_2Einteger_2Eint_of_num V1m))) = (ap c_2Einteger_2Eint_neg \\
& \quad (ap c_2Einteger_2Eint_of_num (ap (ap c_2Earithmetic_2E_2B \\
& \quad V0n) V1m)))))) \wedge (((ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_of_num \\
& \quad V0n)) (ap c_2Einteger_2Eint_of_num V1m)) = (ap c_2Einteger_2Eint_of_num \\
& \quad (ap (ap c_2Earithmetic_2E_2A V0n) V1m))) \wedge (((ap (ap c_2Einteger_2Eint_mul \\
& \quad (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V0n))) \\
& \quad (ap c_2Einteger_2Eint_of_num V1m)) = (ap c_2Einteger_2Eint_neg \\
& \quad (ap c_2Einteger_2Eint_of_num (ap (ap c_2Earithmetic_2E_2A \\
& \quad V0n) V1m)))))) \wedge (((ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_of_num \\
& \quad V0n)) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& \quad V1m))) = (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& \quad (ap (ap c_2Earithmetic_2E_2A V0n) V1m)))))) \wedge (((ap (ap c_2Einteger_2Eint_mul \\
& \quad (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V0n))) \\
& \quad (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V1m))) = \\
& \quad (ap c_2Einteger_2Eint_of_num (ap (ap c_2Earithmetic_2E_2A \\
& \quad V0n) V1m)))))) \wedge (((ap c_2Einteger_2Eint_of_num V0n) = (ap c_2Einteger_2Eint_of_num \\
& \quad V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap c_2Einteger_2Eint_of_num V0n) = (\\
& \quad ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V1m))) \Leftrightarrow \\
& \quad ((V0n = c_2Enum_2E0) \wedge (V1m = c_2Enum_2E0))) \wedge (((ap c_2Einteger_2Eint_neg \\
& \quad (ap c_2Einteger_2Eint_of_num V0n)) = (ap c_2Einteger_2Eint_of_num \\
& \quad V1m)) \Leftrightarrow ((V0n = c_2Enum_2E0) \wedge (V1m = c_2Enum_2E0))) \wedge (((ap c_2Einteger_2Eint_neg \\
& \quad (ap c_2Einteger_2Eint_of_num V0n)) = (ap c_2Einteger_2Eint_neg \\
& \quad (ap c_2Einteger_2Eint_of_num V1m))) \Leftrightarrow (V0n = V1m)) \wedge (((ap c_2Einteger_2Eint_neg \\
& \quad (ap c_2Einteger_2Eint_neg V2x)) = V2x) \wedge ((ap c_2Einteger_2Eint_neg \\
& \quad (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num \\
& \quad c_2Enum_2E0)))))))))
\end{aligned}$$