thm_2Einteger_2EINT__ABS__EQ__ABS (TMdxuiLVAUEU-fireYpBsv96hiMUTHWebMbE)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{3}$$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{4}$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \tag{5}$$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{6}$$

Definition 5 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 6 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 $(ty$

Let $c\_2Einteger\_2Etint\_\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{7}$$

Definition 7 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2$

Definition 8 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 11 *We define* c_2Einteger_2Eint__le *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2E$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{8}$$

Definition 12 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{9}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{10}$$

Definition 13 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \tag{11}$$

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 16 *We define* c_2Einteger_2EABS *to be* $\lambda V0n \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap$ (c_2Ebool_2EC

Definition 17 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 18 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 19 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 20 *We define* c_2Equotient_2E_2D_2D_3E *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f$

Definition 21 *We define* c_2Equotient_2E_3D_3D_3D_3E *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27}$

Definition 22 *We define* c_2Equotient_2EQUOTIENT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda$

Definition 23 *We define* c_2Ecombin_2EW *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x$

Definition 24 *We define* c_2Equotient_2Erespects *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\mathsf{c\_2Ecombin\_2EW}\ A\_27a\ A\_2$

Definition 25 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 26 *We define* c_2Ebool_2ERES__FORALL *to be* $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).($

Definition 27 *We define* c_2Equotient_2EEQUIV *to be* $\lambda A\_27a : \iota.\lambda V0E \in ((2^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E}$

Definition 28 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{18}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{23}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \tag{24}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{25}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p\ V0t1) \Rightarrow (p\ V1t2)) \wedge ((p\ V1t2) \Rightarrow (p\ V0t1)))))) \tag{26}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{28}$$

Assume the following.

$$(\forall V0p \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).\ (\forall V1q \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).\ ((p\ (ap\ (ap\ c\_2Einteger\_2Etint\_\_eq\ V0p)\ V1q)) \Leftrightarrow ((ap\ c\_2Einteger\_2Etint\_\_eq\ V0p) = (ap\ c\_2Einteger\_2Etint\_\_eq\ V1q))))) \tag{29}$$

Assume the following.

$$(\forall V0x \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).\ (\neg(p\ (ap\ (ap\ c\_2Einteger\_2Etint\_\_lt\ V0x)\ V0x)))) \tag{30}$$

Assume the following.

$$(\forall V0x \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).\ (\forall V1y \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).\ (\forall V2z \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).\ (((p\ (ap\ (ap\ c\_2Einteger\_2Etint\_\_lt\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c\_2Einteger\_2Etint\_\_lt\ V1y)\ V2z))) \Rightarrow (p\ (ap\ (ap\ c\_2Einteger\_2Etint\_\_lt\ V0x)\ V2z)))))) \tag{31}$$

Assume the following.

$$(\forall V0x1 \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).\ (\forall V1x2 \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).\ (\forall V2y1 \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).\ (\forall V3y2 \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).\ (((p\ (ap\ (ap\ c\_2Einteger\_2Etint\_\_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c\_2Einteger\_2Etint\_\_eq\ V2y1)\ V3y2))) \Rightarrow ((p\ (ap\ (ap\ c\_2Einteger\_2Etint\_\_lt\ V0x1)\ V2y1)) \Leftrightarrow (p\ (ap\ (ap\ c\_2Einteger\_2Etint\_\_lt\ V1x2)\ V3y2)))))))) \tag{32}$$

Assume the following.

$$(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ ty\_2Einteger\_2Eint)\ c\_2Einteger\_2Etint\_\_eq)\ \mathsf{c\_2Einteger\_2Eint\_\_ABS})\ \mathsf{c\_2Einteger\_2Eint\_\_REP})) \tag{33}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x)) = V0x)) \tag{34}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.\ ((\neg(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ V0x)\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ V1y)\ V0x))))) \tag{35}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2z \in ty\_2Einteger\_2Eint.(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ V0x)\ V1y))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ V1y)\ V2z)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ V0x)\ V2z)))))) \tag{36}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ V0x)\ V1y))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ V1y)\ V0x)))\Leftrightarrow(V0x = V1y)))) \tag{37}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x)))) \tag{38}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ V0x)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))) \tag{39}$$

Assume the following.

$$((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \tag{40}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1y))\Leftrightarrow(V0x = V1y)))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27a)\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a))\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a))\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c. nonempty\ A\_27c\Rightarrow\forall A\_27d.nonempty\ A\_27d\Rightarrow(\forall V0R1\in ((2^{A\_27a})^{A\_27a}).(\forall V1abs1\in (A\_27c^{A\_27a}).(\forall V2rep1\in (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27c)\ V0R1)\ V1abs1)\ V2rep1))\Rightarrow(\forall V3R2\in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2\in (A\_27d^{A\_27b}).(\forall V5rep2\in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2))\Rightarrow(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ (A\_27b^{A\_27a})\ (A\_27d^{A\_27c}))\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ A\_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27a\ A\_27d\ A\_27c\ A\_27b)\ V1abs1)\ V5rep2)))))))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c. nonempty\ A\_27c\Rightarrow\forall A\_27d.nonempty\ A\_27d\Rightarrow(\forall V0R1\in ((2^{A\_27a})^{A\_27a}).(\forall V1abs1\in (A\_27c^{A\_27a}).(\forall V2rep1\in (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27c)\ V0R1)\ V1abs1)\ V2rep1))\Rightarrow(\forall V3R2\in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2\in (A\_27d^{A\_27b}).(\forall V5rep2\in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2))\Rightarrow(\forall V6f\in (A\_27d^{A\_27c}). ((\lambda V7x\in A\_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x\in A\_27a.(ap\ V5rep2\ (ap\ V6f\ (ap\ V1abs1\ V8x)))))))))))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0REL\in ((2^{A\_27a})^{A\_27a}).(\forall V1abs\in (A\_27b^{A\_27a}). (\forall V2rep\in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0REL)\ V1abs)\ V2rep))\Rightarrow(\forall V3x1\in A\_27a.(\forall V4x2\in A\_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2))\Rightarrow(p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep\ (ap\ V1abs\ V4x2))))))))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0R\in ((2^{A\_27a})^{A\_27a}).(\forall V1abs\in (A\_27b^{A\_27a}). (\forall V2rep\in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep))\Rightarrow(\forall V3f\in (2^{A\_27b}).((p\ (ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27b)\ V3f))\Leftrightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27a\ 2\ A\_27b\ 2)\ V1abs)\ (\mathsf{c\_2Ecombin\_2EI}\ 2))\ V3f))))))))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27a}).(\forall V4g \in (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ 2)\ V0R)\ (\mathsf{c\_2Emin\_2E\_3D}\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ V4g)))))))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27c)\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27b^{A\_27a}). (\forall V7g \in (A\_27b^{A\_27a}).(\forall V8x \in A\_27a.(\forall V9y \in A\_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ A\_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \land (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y)))))))))))))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0E \in ((2^{A\_27a})^{A\_27a}). (\forall V1P \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Equotient\_2EEQUIV}\ A\_27a)\ V0E)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27a)\ V1P)))))) \tag{49}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{50}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{51}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{52}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{53}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{54}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{55}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V0p))))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p))) \Rightarrow (p\ V0p))) \tag{64}$$

Theorem 1

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(((ap\ \mathsf{c\_2Einteger\_2EABS}\ V0x) = (ap\ \mathsf{c\_2Einteger\_2EABS}\ V1y)) \Leftrightarrow ((V0x = V1y) \lor (V0x = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1y))))))$$