

thm_2Einteger_2EINT__ABS__MUL (TMFp-
bAQs8aYWgfz2CkJRWur1uC4xP8tEUvu)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (2)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod \\ & \quad A0\ A1) \end{aligned} \quad (3)$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (4)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (5)$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint. (ap (c_2Emin_2E_40 (ty$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\\ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum)} \quad (6)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)} \quad (7)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})})$$

Definition 6 We define $c_2EInteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum\ ty_2Enum)$

Definition 7 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. \lambda V1T2 \in ty_2Einteger_2Eint.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (10)$$

Definition 8 We define c_2Enum_2EO to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 9 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_{\text{min}}(P)$ to be $\lambda P \in 2.\lambda Q \in 2.\text{inj_o } (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 11 We define $c_2Eb0l_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Eb0l_2E_7E))$

Definition 12 We define c_Ebool_E_F_5C to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(\dots)))$

Definition 13 We define $c_{\text{Ebool}} : \lambda A. \exists a : \iota. (\lambda V0P \in (2^{A-27a}).(ap = V))$

Definition 14 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following

$$+ 2\text{Einst}_{\text{even}}(2\text{Einst}_{\text{odd}})^2 \text{num} \in (tu + 2\text{Einst}_{\text{even}}(2\text{Einst}_{\text{odd}})^2\text{Enum}, 2\text{Enum}) \quad (11)$$

Let $\phi_2 E_{\text{int}} \ll \phi_1 E_{\text{int}}$, ϕ_1 can be given. Assume the following:

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})$$

Definition 15 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)} \quad (13)$$

Definition 16 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Etint_lt\ V0T1\ V1T2)$

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ c_2Ebool_2ECOND\ V0t\ V1t1\ V2t2)))$

Definition 18 We define $c_2Einteger_2EABS$ to be $\lambda V0n \in ty_2Einteger_2Eint.(ap\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ V0n))))$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ c_2Enum_2E0) = c_2Enum_2E0)) \quad (14)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2A\ V1n)\ V0m)))) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint.((ap\ c_2Einteger_2Eint_neg\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x)\ V1y)) = (ap\ (ap\ c_2Einteger_2Eint_mul\ (ap\ c_2Einteger_2Eint_neg\ V0x))\ V1y)))) \quad (19)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint.((ap\ c_2Einteger_2Eint_neg\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x)\ V1y)) = (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x)\ (ap\ c_2Einteger_2Eint_neg\ V1y)))))) \quad (20)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
 & ((ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_neg V0x)) \\
 & (ap c_2Einteger_2Eint_neg V1y)) = (ap (ap c_2Einteger_2Eint_mul \\
 & V0x) V1y)))) \\
 \end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & (ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_of_num \\
 & V0m)) (ap c_2Einteger_2Eint_of_num V1n)) = (ap c_2Einteger_2Eint_of_num \\
 & (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))) \\
 \end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in ty_2Einteger_2Eint. ((\exists V1n \in ty_2Enum_2Enum. \\
 & ((V0p = (ap c_2Einteger_2Eint_of_num V1n)) \wedge (\neg(V1n = c_2Enum_2E0)))) \vee \\
 & ((\exists V2n \in ty_2Enum_2Enum. ((V0p = (ap c_2Einteger_2Eint_neg \\
 & (ap c_2Einteger_2Eint_of_num V2n))) \wedge (\neg(V2n = c_2Enum_2E0)))) \vee \\
 & (V0p = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)))))) \\
 \end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Einteger_2EABS (ap c_2Einteger_2Eint_of_num \\
 & V0n)) = (ap c_2Einteger_2Eint_of_num V0n))) \\
 \end{aligned} \tag{24}$$

Assume the following.

$$(\forall V0p \in ty_2Einteger_2Eint. ((ap c_2Einteger_2EABS (ap c_2Einteger_2Eint_neg V0p)) = (ap c_2Einteger_2EABS V0p))) \tag{25}$$

Theorem 1

$$\begin{aligned}
 & (\forall V0p \in ty_2Einteger_2Eint. (\forall V1q \in ty_2Einteger_2Eint. \\
 & ((ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2EABS V0p)) \\
 & (ap c_2Einteger_2EABS V1q)) = (ap c_2Einteger_2EABS (ap (ap c_2Einteger_2Eint_mul \\
 & V0p) V1q)))))) \\
 \end{aligned}$$