

thm_2Einteger_2EINT__DIFFSQ (TMMdJzuZvaSorbFKg8K6rZuZQsAopukR5ma)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Einteger_2E_2tint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2E_2tint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum\ ty_2Eenum_2Eenum)})^{(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum)}) \tag{3}$$

Let $c_2Einteger_2E_2tint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2E_2tint_add \in (((ty_2Epair_2Eprod\ ty_2Eenum_2Eenum\ ty_2Eenum_2Eenum)^{ty_2Epair_2Eprod\ ty_2Eenum_2Eenum})^{(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum)})^{(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum)} \tag{4}$$

Let $ty_2Einteger_2E_2eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2E_2eint \tag{5}$$

Let $c_2Einteger_2E_2eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2E_2eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum\ ty_2Eenum_2Eenum)})^{ty_2Einteger_2E_2eint}) \tag{6}$$

Definition 7 We define $c_2Emin_2E.40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 8 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E.40 (ty$

Let $c_2Einteger_2Eint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum))^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (7)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})} \quad (8)$$

Definition 9 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum$

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)) \quad (9)$$

Definition 10 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint$

Definition 11 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 12 We define $c_2Einteger_2Eint_sub$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (11)$$

Definition 13 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint)^{ty_2Enum_2Enum} \quad (12)$$

Definition 14 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 15 We define $c_2Ecombin_2EK$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0x \in A.27a.(\lambda V1y \in A.27b.V0x)$

Definition 16 We define $c_2Ecombin_2ES$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.(\lambda V0f \in ((A.27c^{A.27b})^{A.27a})$

Definition 17 We define $c_2Ecombin_2EI$ to be $\lambda A.27a : \iota.(ap (ap (c_2Ecombin_2ES A.27a (A.27a^{A.27a}) A$

Definition 18 We define $c_2Equotient_2E.2D.2D.3E$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda A.27d : \iota.\lambda V0f$

Definition 19 We define `c_2Equotient_2E_3D_3D_3D_3E` to be $\lambda A_{27a} : \iota. \lambda A_{27b} : \iota. \lambda V_0R_1 \in ((2^{A_{27a}})^{A_{27a}})$

Definition 20 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V_0t_1 \in 2. (\lambda V_1t_2 \in 2. (ap (c_2Ebool_2E_{21} 2) (\lambda V_2t \in$

Definition 21 We define `c_2Equotient_2EQUOTIENT` to be $\lambda A_{27a} : \iota. \lambda A_{27b} : \iota. \lambda V_0R \in ((2^{A_{27a}})^{A_{27a}})$

Definition 22 We define `c_2Ecombin_2EW` to be $\lambda A_{27a} : \iota. \lambda A_{27b} : \iota. (\lambda V_0f \in ((A_{27b}^{A_{27a}})^{A_{27a}}). (\lambda V_1x$

Definition 23 We define `c_2Equotient_2Erespects` to be $\lambda A_{27a} : \iota. \lambda A_{27b} : \iota. (c_2Ecombin_2EW A_{27a} A_{27b})$

Definition 24 We define `c_2Ebool_2EIN` to be $\lambda A_{27a} : \iota. (\lambda V_0x \in A_{27a}. (\lambda V_1f \in (2^{A_{27a}}). (ap V_1f V_0x)))$

Definition 25 We define `c_2Ebool_2ERES_FORALL` to be $\lambda A_{27a} : \iota. (\lambda V_0p \in (2^{A_{27a}}). (\lambda V_1m \in (2^{A_{27a}}).$

Definition 26 We define `c_2Equotient_2EEQUIV` to be $\lambda A_{27a} : \iota. \lambda V_0E \in ((2^{A_{27a}})^{A_{27a}}). (ap (c_2Ebool_2E$

Assume the following.

$$True \tag{13}$$

Assume the following.

$$\begin{aligned} & (\forall V_0t \in 2. (((True \Rightarrow (p V_0t)) \Leftrightarrow (p V_0t)) \wedge (((p V_0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V_0t)) \Leftrightarrow True) \wedge (((p V_0t) \Rightarrow (p V_0t)) \Leftrightarrow True) \wedge ((\\ & (p V_0t) \Rightarrow False) \Leftrightarrow (\neg (p V_0t)))))) \end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned} \forall A_{27a}. nonempty A_{27a} \Rightarrow & (\forall V_0x \in A_{27a}. (\forall V_1y \in \\ & A_{27a}. ((V_0x = V_1y) \Leftrightarrow (V_1y = V_0x)))) \end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned} & (\forall V_0t \in 2. (((True \Leftrightarrow (p V_0t)) \Leftrightarrow (p V_0t)) \wedge (((p V_0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V_0t)) \wedge (((False \Leftrightarrow (p V_0t)) \Leftrightarrow (\neg (p V_0t))) \wedge (((p V_0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V_0t)))))) \end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned} & (\forall V_0t_1 \in 2. (\forall V_1t_2 \in 2. (\forall V_2t_3 \in 2. (((p V_0t_1) \Rightarrow \\ & ((p V_1t_2) \Rightarrow (p V_2t_3))) \Leftrightarrow (((p V_0t_1) \wedge (p V_1t_2)) \Rightarrow (p V_2t_3)))))) \end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned} \forall A_{27a}. nonempty A_{27a} \Rightarrow & (\forall V_0x \in A_{27a}. ((ap (c_2Ecombin_2EI \\ & A_{27a}) V_0x) = V_0x)) \end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned} & (\forall V_0p \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\ & (\forall V_1q \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\ & ((p (ap (ap c_2Einteger_2Etint_eq V_0p) V_1q)) \Leftrightarrow ((ap c_2Einteger_2Etint_eq \\ & V_0p) = (ap c_2Einteger_2Etint_eq V_1q)))))) \end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1q \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \quad (20) \\
& ((V0p = V1q) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0p)\ V1q))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1y \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& ((ap\ (ap\ c_2Einteger_2Etint_add\ V0x)\ V1y) = (ap\ (ap\ c_2Einteger_2Etint_add\ V1y)\ V0x)))) \quad (21)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1y \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& ((ap\ (ap\ c_2Einteger_2Etint_mul\ V0x)\ V1y) = (ap\ (ap\ c_2Einteger_2Etint_mul\ V1y)\ V0x)))) \quad (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1y \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V2z \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& ((ap\ (ap\ c_2Einteger_2Etint_add\ V0x)\ (ap\ (ap\ c_2Einteger_2Etint_add\ V1y)\ V2z)) = (ap\ (ap\ c_2Einteger_2Etint_add\ (ap\ (ap\ c_2Einteger_2Etint_add\ V0x)\ V1y))\ V2z)))) \quad (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1y \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V2z \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& ((ap\ (ap\ c_2Einteger_2Etint_mul\ V0x)\ (ap\ (ap\ c_2Einteger_2Etint_add\ V1y)\ V2z)) = (ap\ (ap\ c_2Einteger_2Etint_add\ (ap\ (ap\ c_2Einteger_2Etint_mul\ V0x)\ V1y))\ (ap\ (ap\ c_2Einteger_2Etint_mul\ V0x)\ V2z)))) \quad (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1x2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V2y1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V3y2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (((p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V2y1)\ V3y2))) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ (ap\ (ap\ c_2Einteger_2Etint_add\ V0x1)\ V2y1))\ (ap\ (ap\ c_2Einteger_2Etint_add\ V1x2)\ V3y2)))))) \quad (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1x2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V2y1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V3y2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (((p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V2y1)\ V3y2))) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ (ap\ (ap\ c_2Einteger_2Etint_mul\ V0x1)\ V2y1))\ (ap\ (ap\ c_2Einteger_2Etint_mul\ V1x2)\ V3y2))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)\ ty_2Einteger_2Eint)\ c_2Einteger_2Etint_eq)\ c_2Einteger_2Eint_ABS)\ c_2Einteger_2Eint_REP))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. ((ap\ (ap\ c_2Einteger_2Eint_mul\ (ap\ (ap\ c_2Einteger_2Eint_add\ V0x)\ V1y))\ V2z) = (ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x)\ V2z))\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V1y)\ V2z))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (((ap\ (ap\ c_2Einteger_2Eint_add\ V0x)\ V1y) = V1y) \Leftrightarrow (V0x = (ap\ c_2Einteger_2Eint_of_num\ c_2Eenum_2E0))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (((ap\ (ap\ c_2Einteger_2Eint_add\ V0x)\ V1y) = (ap\ c_2Einteger_2Eint_of_num\ c_2Eenum_2E0)) \Leftrightarrow (V0x = (ap\ c_2Einteger_2Eint_neg\ V1y))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap\ c_2Einteger_2Eint_neg\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x)\ V1y)) = (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x)\ (ap\ c_2Einteger_2Eint_neg\ V1y))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27a)\ (c_2Emin_2E_3D\ A_27a))\ (c_2Ecombin_2EI\ A_27a))\ (c_2Ecombin_2EI\ A_27a)))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad (A_27b^{A_27a})\ (A_27d^{A_27c}))\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\
& \quad A_27a\ A_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27c \\
& \quad A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27a\ A_27d\ A_27c\ A_27b)\ V1abs1)\ V5rep2))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x \in A_27b.(\forall V4y \in \\
& \quad A_27b.((V3x = V4y) \Leftrightarrow (p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3x))\ (ap\ V2rep\ V4y))))))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& \quad A_27a.(\forall V5y1 \in A_27a.(\forall V6y2 \in A_27a.(((p\ (ap\ (ap\ V0R \\
& \quad V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2))))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27d^{A_27c}). \\
& \quad ((\lambda V7x \in A_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x))))))))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0REL \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& \quad A_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2))))))))))))) \\
& \hspace{15em} (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27b}).((p\ (\\
& \quad ap\ (c_2Ebool_2E_21\ A_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2ERespects\ A_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b\ 2)\ V1abs)\ (c_2Ecombin_2EI \\
& \quad 2))\ V3f))))))))))))) \\
& \hspace{15em} (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27a}).(\forall V4g \in \\
& \quad (2^{A_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad 2)\ V0R)\ (c_2Emin_2E_3D\ 2)\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2ERespects\ A_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (\\
& \quad ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2ERespects \\
& \quad A_27a\ 2)\ V0R))\ V4g))))))))))))) \\
& \hspace{15em} (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b^{A_27a}). \\
& \quad (\forall V7g \in (A_27b^{A_27a}).(\forall V8x \in A_27a.(\forall V9y \in \\
& \quad A_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))))))))))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V_0 E \in ((2^{A_{27a}})^{A_{27a}}). \\
& (\forall V_1 P \in (2^{A_{27a}}). ((p (ap (c_2Equotient_2EEQUIV } A_{27a} \\
V_0 E)) \Rightarrow ((p (ap (ap (c_2Ebool_2ERES_FORALL } A_{27a} (ap (c_2Equotient_2Erespects \\
A_{27a} 2) V_0 E)) V_1 P)) \Leftrightarrow (p (ap (c_2Ebool_2E.21 } A_{27a} V_1 P))))))
\end{aligned} \tag{41}$$

Theorem 1

$$\begin{aligned}
& (\forall V_0 x \in ty_2Einteger_2Eint. (\forall V_1 y \in ty_2Einteger_2Eint. \\
& ((ap (ap c_2Einteger_2Eint_mul (ap (ap c_2Einteger_2Eint_add \\
V_0 x) V_1 y)) (ap (ap c_2Einteger_2Eint_sub V_0 x) V_1 y)) = (ap (ap c_2Einteger_2Eint_sub \\
(ap (ap c_2Einteger_2Eint_mul V_0 x) V_0 x)) (ap (ap c_2Einteger_2Eint_mul \\
V_1 y) V_1 y))))))
\end{aligned}$$