

thm_2Einteger_2EINT__DIV__CALCULATE (TMTG5kKoBdKGwTaKp5JaQKaS8taZbTKzM8R)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \tag{1}$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Eenum_2Eenum^{ty_2Eenum_2Eenum})^{ty_2Eenum_2Eenum}) \tag{2}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{3}$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{4}$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum\ ty_2Eenum_2Eenum)})^{ty_2Einteger_2Eint}) \tag{5}$$

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty$

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum) (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)) \quad (6)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)) \quad (7)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})} \quad (8)$$

Definition 8 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$

Definition 9 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint_neg$

Let $c_2Einteger_2Eint_div : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_div \in ((ty_2Einteger_2Eint)^{ty_2Einteger_2Eint})^{ty_2Einteger_2Eint} \quad (9)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (11)$$

Definition 10 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint)^{ty_2Enum_2Enum} \quad (12)$$

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_neg V0x)) = V0x)) \quad (13)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(((ap c_2Einteger_2Eint_neg V0x) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \Leftrightarrow (V0x = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)))) \quad (14)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Einteger_2Eint_of_num\ V0m) = (ap\ c_2Einteger_2Eint_of_num \\
& \quad V1n)) \Leftrightarrow (V0m = V1n)))) \\
& \tag{15}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (\neg(V1m = c_2Enum_2E0)) \Rightarrow ((ap\ (ap\ c_2Einteger_2Eint_div\ (ap\ c_2Einteger_2Eint_of_num \\
& \quad V0n))\ (ap\ c_2Einteger_2Eint_of_num\ V1m)) = (ap\ c_2Einteger_2Eint_of_num \\
& \quad (ap\ (ap\ c_2Earithmetic_2EDIV\ V0n)\ V1m)))))) \\
& \tag{16}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Einteger_2Eint. (\forall V1q \in ty_2Einteger_2Eint. \\
& ((\neg(V1q = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))) \Rightarrow ((ap \\
& (ap\ c_2Einteger_2Eint_div\ V0p)\ (ap\ c_2Einteger_2Eint_neg\ V1q)) = \\
& (ap\ (ap\ c_2Einteger_2Eint_div\ (ap\ c_2Einteger_2Eint_neg\ V0p)) \\
& \quad V1q)))))) \\
& \tag{17}
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
& ((\neg(V1m = c_2Enum_2E0)) \Rightarrow ((ap\ (ap\ c_2Einteger_2Eint_div\ (ap\ c_2Einteger_2Eint_of_num \\
& \quad V0n))\ (ap\ c_2Einteger_2Eint_of_num\ V1m)) = (ap\ c_2Einteger_2Eint_of_num \\
& \quad (ap\ (ap\ c_2Earithmetic_2EDIV\ V0n)\ V1m)))))) \wedge ((\forall V2p \in ty_2Einteger_2Eint. \\
& (\forall V3q \in ty_2Einteger_2Eint. ((\neg(V3q = (ap\ c_2Einteger_2Eint_of_num \\
& \quad c_2Enum_2E0))) \Rightarrow ((ap\ (ap\ c_2Einteger_2Eint_div\ V2p)\ (ap\ c_2Einteger_2Eint_neg \\
& \quad V3q)) = (ap\ (ap\ c_2Einteger_2Eint_div\ (ap\ c_2Einteger_2Eint_neg \\
& \quad V2p))\ V3q)))))) \wedge ((\forall V4m \in ty_2Enum_2Enum. (\forall V5n \in ty_2Enum_2Enum. \\
& (((ap\ c_2Einteger_2Eint_of_num\ V4m) = (ap\ c_2Einteger_2Eint_of_num \\
& \quad V5n)) \Leftrightarrow (V4m = V5n))) \wedge ((\forall V6x \in ty_2Einteger_2Eint. (((ap \\
& \quad c_2Einteger_2Eint_neg\ V6x) = (ap\ c_2Einteger_2Eint_of_num \\
& \quad c_2Enum_2E0)) \Leftrightarrow (V6x = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)))) \wedge \\
& (\forall V7x \in ty_2Einteger_2Eint. ((ap\ c_2Einteger_2Eint_neg \\
& \quad (ap\ c_2Einteger_2Eint_neg\ V7x)) = V7x))))))
\end{aligned}$$