

thm_2Einteger_2EINT__DIV__FORALL__P
 (TMYFyY-
 HBBH5UfvG4CZPU3eW1M7q42jWMvrX)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

Let $ty_2Einteger_2E_int : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2E_int \tag{1}$$

Let $c_2Einteger_2E_int_mod : \iota$ be given. Assume the following.

$$c_2Einteger_2E_int_mod \in ((ty_2Einteger_2E_int^{ty_2Einteger_2E_int})^{ty_2Einteger_2E_int}) \tag{2}$$

Let $c_2Einteger_2E_int_div : \iota$ be given. Assume the following.

$$c_2Einteger_2E_int_div \in ((ty_2Einteger_2E_int^{ty_2Einteger_2E_int})^{ty_2Einteger_2E_int}) \tag{3}$$

Let $ty_2Eenum_2E_enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2E_enum \tag{4}$$

Let $ty_2Epair_2E_eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2E_eprod\ A0\ A1) \tag{5}$$

Let $c_2Einteger_2E_int_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2E_int_REP_CLASS \in ((2^{(ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum)})^{ty_2Einteger_2E_int}) \tag{6}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. P\ x) \mathbf{then} (the\ (\lambda x. x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)} \quad (14)$$

Definition 14 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 16 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (17)$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A.27a. (p\ (ap\ V1P\ V3x))) \vee (p\ V0Q)))))) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A.27a. (p\ (ap\ V1Q\ V3x)))))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B)) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (25)$$

Assume the following.

$$(\forall V0q \in ty_2Einteger_2Eint. ((\neg (V0q = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))) \Rightarrow (\forall V1p \in ty_2Einteger_2Eint. (((V1p = (ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ (ap\ c_2Einteger_2Eint_mul\ (ap\ (ap\ c_2Einteger_2Eint_div\ V1p)\ V0q))\ V0q))\ (ap\ (ap\ c_2Einteger_2Eint_mod\ V1p)\ V0q))) \wedge (p\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ 2)\ (ap\ (ap\ c_2Einteger_2Eint_lt\ V0q)\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)))\ (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ c_2Einteger_2Eint_lt\ V0q)\ (ap\ (ap\ c_2Einteger_2Eint_mod\ V1p)\ V0q)))\ (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ (ap\ c_2Einteger_2Eint_mod\ V1p)\ V0q))\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))))\ (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))\ (ap\ (ap\ c_2Einteger_2Eint_mod\ V1p)\ V0q)))\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ (ap\ c_2Einteger_2Eint_mod\ V1p)\ V0q)))))))))) \quad (26)$$

Assume the following.

$$(\forall V0i \in ty_2Einteger_2Eint. (\forall V1j \in ty_2Einteger_2Eint. (\forall V2q \in ty_2Einteger_2Eint. ((\exists V3r \in ty_2Einteger_2Eint. ((V0i = (ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V2q)\ V1j))\ V3r)) \wedge (p\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ 2)\ (ap\ c_2Einteger_2Eint_lt\ V1j)\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)))\ (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ c_2Einteger_2Eint_lt\ V1j)\ V3r))\ (ap\ (ap\ c_2Einteger_2Eint_le\ V3r)\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))))\ (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))\ V3r))\ (ap\ (ap\ c_2Einteger_2Eint_lt\ V3r)\ V1j)))))) \Rightarrow ((ap\ (ap\ c_2Einteger_2Eint_div\ V0i)\ V1j) = V2q)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (32)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (37)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (\forall V3s \in \\
& 2. (((p V0p) \Leftrightarrow (p (ap (ap (ap (c_2Ebool_2ECOND 2) V1q) V2r) V3s))) \Leftrightarrow \\
& (((p V0p) \vee ((p V1q) \vee (\neg(p V3s)))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge \\
& (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V3s)))) \wedge (((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))) \wedge ((p V1q) \vee ((p V3s) \vee (\neg(p V0p))))))))))))) \\
& \tag{38}
\end{aligned}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{39}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{40}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{41}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{42}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{43}$$

Theorem 1

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Einteger_2Eint}). (\forall V1x \in ty_2Einteger_2Eint. \\
& (\forall V2c \in ty_2Einteger_2Eint. ((\neg(V2c = (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))) \Rightarrow ((p (ap V0P (ap (ap c_2Einteger_2Eint_div V1x) \\
& V2c))) \Leftrightarrow (\forall V3k \in ty_2Einteger_2Eint. (\forall V4r \in ty_2Einteger_2Eint. \\
& (((V1x = (ap (ap c_2Einteger_2Eint_add (ap (ap c_2Einteger_2Eint_mul \\
& V3k) V2c)) V4r)) \wedge (((p (ap (ap c_2Einteger_2Eint_lt V2c) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))) \wedge ((p (ap (ap c_2Einteger_2Eint_lt V4r)) \wedge \\
& (p (ap (ap c_2Einteger_2Eint_le V4r) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)))))) \vee ((\neg(p (ap (ap c_2Einteger_2Eint_lt V2c) (ap \\
& c_2Einteger_2Eint_of_num c_2Enum_2E0)))) \wedge ((p (ap (ap c_2Einteger_2Eint_le \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0) V4r)) \wedge (p (ap (ap \\
& c_2Einteger_2Eint_lt V4r) V2c)))))) \Rightarrow (p (ap V0P V3k)))))))))
\end{aligned}$$