

# thm\_2Einteger\_2EINT\_\_DOUBLE (TM- PRrT8YE5UVtpqviLVS3UyKnoHjP1DPAx3)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 3** We define  $c\_2Ebool\_2E2$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (c\_2Enum\_2ESUC\_REP\ m))$

Let  $c\_2Earithmetic\_2E2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 6** We define  $c\_2Earithmic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmic\_2EBIT2) n)$ .  
Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (7)$$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty ty\_2Einteger\_2Eint \quad (8)$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint}) \quad (9)$$

**Definition 7** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  then  $(the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E40) a)$ .

Let  $c\_2Einteger\_2Eint\_mul : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_mul \in (((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{ty\_2Einteger\_2Eint})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)}) \quad (10)$$

Let  $c\_2Einteger\_2Eint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)}) \quad (11)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})} \quad (12)$$

**Definition 9** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)$ .

**Definition 10** We define  $c\_2Einteger\_2Eint\_mul$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint$ .

**Definition 11** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 12** We define  $c\_2Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmic\_2EBIT1) n)$ .

**Definition 13** We define  $c\_2Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Einteger\_2Eint\_add : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_add \in (((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{ty\_2Einteger\_2Eint})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)}) \quad (13)$$

**Definition 14** We define  $c\_Einteger\_2Eint\_add$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint$

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (14)$$

Assume the following.

$$\begin{aligned} & ((ap\ c\_2Earithmic\_2ENUMERAL\ (ap\ c\_2Earithmic\_2EBIT2\ c\_2Earithmic\_2EZERO)) = \\ & \quad (ap\ c\_2Enum\_2ESUC\ (ap\ c\_2Earithmic\_2ENUMERAL\ (ap\ c\_2Earithmic\_2EBIT1 \\ & \quad \quad c\_2Earithmic\_2EZERO)))) \end{aligned} \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ c\_2Einteger\_2Eint\_mul \\ & \quad (ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ c\_2Earithmic\_2ENUMERAL \\ & \quad \quad (ap\ c\_2Earithmic\_2EBIT1\ c\_2Earithmic\_2EZERO))))\ V0x) = V0x)) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint. \\ & \quad (\forall V2z \in ty\_2Einteger\_2Eint.((ap\ (ap\ c\_2Einteger\_2Eint\_mul \\ & \quad (ap\ (ap\ c\_2Einteger\_2Eint\_add\ V0x\ V1y))\ V2z) = (ap\ (ap\ c\_2Einteger\_2Eint\_add \\ & \quad \quad (ap\ (ap\ c\_2Einteger\_2Eint\_mul\ V0x\ V2z))\ (ap\ (ap\ c\_2Einteger\_2Eint\_mul \\ & \quad \quad \quad V1y\ V2z)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum.((ap\ c\_2Einteger\_2Eint\_of\_num \\ & \quad (ap\ c\_2Enum\_2ESUC\ V0n)) = (ap\ (ap\ c\_2Einteger\_2Eint\_add\ (ap\ c\_2Einteger\_2Eint\_of\_num \\ & \quad \quad V0n))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ c\_2Earithmic\_2ENUMERAL \\ & \quad \quad \quad (ap\ c\_2Earithmic\_2EBIT1\ c\_2Earithmic\_2EZERO)))))) \end{aligned} \quad (20)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ c\_2Einteger\_2Eint\_add \\ & \quad V0x)\ V0x) = (ap\ (ap\ c\_2Einteger\_2Eint\_mul\ (ap\ c\_2Einteger\_2Eint\_of\_num \\ & \quad \quad (ap\ c\_2Earithmic\_2ENUMERAL\ (ap\ c\_2Earithmic\_2EBIT2\ c\_2Earithmic\_2EZERO)))) \\ & \quad \quad \quad V0x))) \end{aligned}$$