thm_2Einteger_2EINT__EXP__CALCULATE (TMagmfXLyK3JihtYv2x2cMGa3dU2creUetR)

October 26, 2020

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (3)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (5)$$

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

Definition 6 *We define* $\mathsf{c\_2Earithmetic\_2EBIT2}$ *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 7 *We define* $\mathsf{c\_2Ebool\_2EF}$ *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 *We define* $\mathsf{c\_2Emin\_2E\_3D\_3D\_3E}$ *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 9 *We define* $\mathsf{c\_2Ebool\_2E\_7E}$ *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{7}$$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{8}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{9}$$

Definition 10 *We define* $\mathsf{c\_2Emin\_2E\_40}$ *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 11 *We define* $\mathsf{c\_2Einteger\_2Eint\_\_REP}$ *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (t$

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{10}$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{11}$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \tag{12}$$

Definition 12 *We define* $\mathsf{c\_2Einteger\_2Eint\_\_ABS}$ *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 13 *We define* $\mathsf{c\_2Einteger\_2Eint\_\_mul}$ *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteg$

Definition 14 *We define* $\mathsf{c\_2Earithmetic\_2EZERO}$ *to be* $\mathsf{c\_2Enum\_2E0}$.

Definition 15 *We define* $\mathsf{c\_2Earithmetic\_2EBIT1}$ *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 16 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{13}$$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \tag{15}$$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{16}$$

Definition 17 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Let $c\_2Einteger\_2Eint\_\_exp : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_exp \in ((ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum})^{ty\_2Einteger\_2Eint}) \tag{17}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{18}$$

Definition 18 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Assume the following.

$$(((p\ (ap\ c\_2Earithmetic\_2EEVEN\ \mathsf{c\_2Enum\_2E0})) \Leftrightarrow True) \wedge (\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ c\_2Earithmetic\_2EEVEN\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n))) \Leftrightarrow (\neg(p\ (ap\ c\_2Earithmetic\_2EEVEN\ V0n)))))) \tag{19}$$

Assume the following.

$$(((p\ (ap\ c\_2Earithmetic\_2EODD\ \mathsf{c\_2Enum\_2E0})) \Leftrightarrow False) \wedge (\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n))) \Leftrightarrow (\neg(p\ (ap\ c\_2Earithmetic\_2EODD\ V0n)))))) \tag{20}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))) \tag{21}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ c\_2Earithmetic\_2EEVEN\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))\Leftrightarrow((p\ (ap\ c\_2Earithmetic\_2EEVEN\ V0m))\Leftrightarrow(p\ (ap\ c\_2Earithmetic\_2EEVEN\ V1n)))))) \tag{22}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))\Leftrightarrow(\neg((p\ (ap\ c\_2Earithmetic\_2EODD\ V0m))\Leftrightarrow(p\ (ap\ c\_2Earithmetic\_2EODD\ V1n))))))) \tag{23}$$

Assume the following.

$$True \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow(p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge(((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{26}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge((\neg False)\Leftrightarrow True))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{29}$$

Assume the following.

$$((\forall V0p \in ty\_2Einteger\_2Eint.((ap\ (ap\ c\_2Einteger\_2Eint\_\_exp\ V0p)\ \mathsf{c\_2Enum\_2E0}) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO})))))\wedge(\forall V1p \in ty\_2Einteger\_2Eint.(\forall V2n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Einteger\_2Eint\_\_exp\ V1p)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V1p)\ (ap\ (ap\ c\_2Einteger\_2Eint\_\_exp\ V1p)\ V2n)))))) \tag{30}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Einteger\_2Eint\_\_exp\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n))\ V1m) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ V0n)\ V1m))))) \tag{31}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ c\_2Earithmetic\_2EEVEN\ V0n))\Rightarrow((ap\ (ap\ c\_2Einteger\_2Eint\_\_exp\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)))\ V0n) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ V1m)\ V0n))))\wedge((p\ (ap\ c\_2Earithmetic\_2EODD\ V0n))\Rightarrow((ap\ (ap\ c\_2Einteger\_2Eint\_\_exp\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)))\ V0n) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ V1m)\ V0n)))))))) \tag{32}$$

Theorem 1

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1n \in ty\_2Enum\_2Enum. (\forall V2m \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Einteger\_2Eint\_\_exp\ V0p)\ \mathsf{c\_2Enum\_2E0}) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge(((ap\ (ap\ c\_2Einteger\_2Eint\_\_exp\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n))\ V2m) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ V1n)\ V2m)))\wedge(((ap\ (ap\ c\_2Einteger\_2Eint\_\_exp\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2m))) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ V1n)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2m)))))))\wedge((ap\ (ap\ c\_2Einteger\_2Eint\_\_exp\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2m))) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ V1n)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2m)))))))))))$$