thm_2Einteger_2EINT_EXP_SUBTRACT_EXPONENTS (TMUBp9sXLxanTj1mtuSfzxnhfLosEfLwYTb)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t)).$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{3}$$

Let $c\_2Einteger\_2Eint\_\_div : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_div \in ((ty\_2Einteger\_2Eint^{ty\_2Einteger\_2Eint})^{ty\_2Einteger\_2Eint}) \tag{4}$$

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Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod$$
$$A0\ A1)$$
$$(5)$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2}$$
$$(6)$$

Definition 9 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 $(ty.$

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty}$$
$$(7)$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum}$$
$$(8)$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}$$
$$(9)$$

Definition 10 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty.$

Definition 11 *We define* c_2Einteger_2Eint__mul *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteg$

Let $c\_2Einteger\_2Eint\_\_exp : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_exp \in ((ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum})^{ty\_2Einteger\_2Eint})$$
$$(10)$$

Let $c\_2Einteger\_2Eint\_\_mod : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_mod \in ((ty\_2Einteger\_2Eint^{ty\_2Einteger\_2Eint})^{ty\_2Einteger\_2Eint})$$
$$(11)$$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (12)$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (13)$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (14)$$

Definition 12 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E$\ V0t)\ $c_2Ebool_2E

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (15)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (16)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (17)$$

Definition 13 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 14 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 15 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 16 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 17 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

Definition 18 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (18)$$

Definition 19 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Definition 20 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(19)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(20)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(21)$$

Definition 21 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 22 *We define* c_2Enumeral_2EiiSUC *to be* $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Enum_2ESUC\ (ap$

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Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

$$(22)$$

Definition 23 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 24 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmeti$

Definition 25 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmeti$

Definition 26 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 27 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Assume the following.

$$((\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$\mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}))\wedge(\forall V1m \in ty\_2Enum\_2Enum.$$
$$(\forall V2n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ ($$
$$ap\ \mathsf{c\_2Enum\_2ESUC}\ V1m))\ V2n) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)$$
$$(ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V2n))\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1m)\ V2n)))))))$$

$$(23)$$

Assume the following.

$$((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})) =$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ \mathsf{c\_2Enum\_2E0}))$$

$$(24)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m)\wedge(((ap\ ($$
$$ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m)\wedge(((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0m)\ V1n)))\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))))))))$$

$$(25)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V1n)\ V0m))))$$

$$(26)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p)))))$$

$$(27)$$

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Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\
(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V0n))\ (ap\\
\textsf{c\_2Enum\_2ESUC}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\\
V1m)))))) \tag{28}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\\
(ap\ \textsf{c\_2Enum\_2ESUC}\ V0m))\ V1n))))))$$

$$\tag{29}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\\
\textsf{c\_2Enum\_2E0})\ V0n))) \tag{30}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\\
\textsf{c\_2Enum\_2E0})\ V0m) = \textsf{c\_2Enum\_2E0})\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2D\\
V0m)\ \textsf{c\_2Enum\_2E0}) = V0m)))$$

$$\tag{31}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\
(ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V0n))\ (ap\ \textsf{c\_2Enum\_2ESUC}\\
V1m)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)\ V1m))))$$

$$\tag{32}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \textsf{c\_2Enum\_2E0})\ V0m) = \textsf{c\_2Enum\_2E0})\wedge\\
(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \textsf{c\_2Enum\_2E0}) = \textsf{c\_2Enum\_2E0})\wedge\\
((((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\\
(ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m)\wedge\\
(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\\
(ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO}))) = V0m)\wedge(\\
((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\\
(ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\\
V1n))\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V1n)) =\\
(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\\
V0m)\ V1n)))))))))))$$

$$\tag{33}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\\
V0m)\ V1n))\wedge(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))\Rightarrow(p\ (\\
ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p)))))) \tag{34}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0m)\ V0m))) \tag{35}$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0c)$$
$$V0c) = \mathsf{c\_2Enum\_2E0})) \tag{36}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0m)\ V2p))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))))) \tag{37}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V0m))))) \tag{38}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(\neg(V0m = V1n)) \Leftrightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V0m))\ V1n)) \lor (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n))\ V0m)))))) \tag{39}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap$$
$$c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO})))\ V0n))) \tag{40}$$

Assume the following.

$$(\forall V0q \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EDIV\ V0q)$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ \mathsf{c\_2Enum\_2E0})) = V0q)) \tag{41}$$

Assume the following.
$$True \tag{42}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p$$
$$V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{43}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{44}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \atop A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{46}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \tag{47}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{48}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow \atop (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \atop (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{49}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge \atop (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \atop (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{50}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow \atop True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\atop (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t))))))))) \tag{51}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \atop ((\neg False) \Leftrightarrow True))) \tag{52}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \atop True)) \tag{53}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \atop A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{54}$$

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Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow$$
$$(p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\quad (55)$$
$$p\ V0t)))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in$$
$$A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)$$
$$V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})$$
$$V0t1)\ V1t2) = V1t2))))$$

$$(56)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ($$
$$(p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (57)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee$$
$$(p\ V0A))))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg($$
$$p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))))$$
$$(59)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee$$
$$(p\ V1B))))) \quad (60)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (61)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow$$
$$((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (62)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in$$
$$2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \quad (63)$$
$$(((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27))))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.$$
$$(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.$$
$$(\forall V5y\_27 \in A\_27a.(((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge$$
$$((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)$$
$$V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)$$
$$V5y\_27)))))))))$$

$$(64)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V1y) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})) \Leftrightarrow ((V0x = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \vee$$
$$(V1y = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))))$$

$$(65)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$(\neg(V1m = \mathsf{c\_2Enum\_2E0})) \Rightarrow ((ap\ (ap\ c\_2Einteger\_2Eint\_\_div\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$V0n))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ (ap\ c\_2Earithmetic\_2EDIV\ V0n)\ V1m))))))$$

$$(66)$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.((\neg(V0p = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))) \Rightarrow ((ap\ (ap\ c\_2Einteger\_2Eint\_\_div\ V0p)\ V0p) = (ap$$
$$c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap$$
$$\mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))$$

$$(67)$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1q \in ty\_2Einteger\_2Eint.$$
$$(\forall V2k \in ty\_2Einteger\_2Eint.(((\neg(V1q = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))) \wedge ((ap\ (ap\ c\_2Einteger\_2Eint\_\_mod\ V0p)\ V1q) = (ap$$
$$c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \Rightarrow ((ap\ (ap\ c\_2Einteger\_2Eint\_\_div$$
$$(ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2k)\ V0p))\ V1q) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}$$
$$V2k)\ (ap\ (ap\ c\_2Einteger\_2Eint\_\_div\ V0p)\ V1q)))))))$$

$$(68)$$

Assume the following.

$$((\forall V0p \in ty\_2Einteger\_2Eint.((ap\ (ap\ c\_2Einteger\_2Eint\_\_exp$$
$$V0p)\ \mathsf{c\_2Enum\_2E0}) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \wedge (\forall V1p \in$$
$$ty\_2Einteger\_2Eint.(\forall V2n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Einteger\_2Eint\_\_exp$$
$$V1p)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V1p)$$
$$(ap\ (ap\ c\_2Einteger\_2Eint\_\_exp\ V1p)\ V2n))))))$$

$$(69)$$

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Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1n \in ty\_2Enum\_2Enum.$$
$$(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_exp}\ V0p)\ V1n) = (ap\ \mathsf{c\_2Einteger\_2Eint\_of\_num}$$
$$\mathsf{c\_2Enum\_2E0})) \Leftrightarrow ((V0p = (ap\ \mathsf{c\_2Einteger\_2Eint\_of\_num}\ \mathsf{c\_2Enum\_2E0})) \wedge$$
$$(\neg(V1n = \mathsf{c\_2Enum\_2E0}))))))$$

$$(70)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Einteger\_2Eint.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1n)\ V0m)) \wedge (\neg(V2p = (ap\ \mathsf{c\_2Einteger\_2Eint\_of\_num}\ \mathsf{c\_2Enum\_2E0})))) \Rightarrow$$
$$((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_mod}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_exp}$$
$$V2p)\ V0m))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_exp}\ V2p)\ V1n)) = (ap\ \mathsf{c\_2Einteger\_2Eint\_of\_num}$$
$$\mathsf{c\_2Enum\_2E0})))))))$$

$$(71)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = \mathsf{c\_2Enum\_2E0})))$$

$$(72)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})) \wedge$$
$$(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n)))))$$

$$(73)$$

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
c_2Enum_2E0) $V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)$ c_2Enum_2E0) $= V1n))\wedge((\forall V2n \in$
$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$(ap$ c_2Earithmetic_2ENUMERAL $V2n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V3m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Enumeral_2EiZ $(ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2A$ c_2Enum_2E0) $V4n) =$ c_2Enum_2E0))$\wedge$
$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V5n)$ c_2Enum_2E0) $=$ c_2Enum_2E0))$\wedge((\forall V6n \in ty\_2Enum\_2Enum.$
$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($
ap c_2Earithmetic_2ENUMERAL $V6n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V7m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
c_2Enum_2E0) $V8n) =$ c_2Enum_2E0))$\wedge((\forall V9n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)$ c_2Enum_2E0) $= V9n))\wedge((\forall V10n \in$
$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$(ap$ c_2Earithmetic_2ENUMERAL $V10n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V11m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$
c_2Enum_2E0) $(ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1
$V12n))) =$ c_2Enum_2E0))$\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2EEXP$ c_2Enum_2E0) $(ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Earithmetic_2EBIT2 $V13n))) =$ c_2Enum_2E0))$\wedge((\forall V14n \in$
$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)$ c_2Enum_2E0) $=$
$(ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))$\wedge$
$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap$ c_2Earithmetic_2ENUMERAL $V15n))$
$(ap$ c_2Earithmetic_2ENUMERAL $V16m)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap$ c_2Enum_2ESUC
c_2Enum_2E0) $= (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1
c_2Earithmetic_2EZERO)))$\wedge((\forall V17n \in ty\_2Enum\_2Enum.($
$(ap$ c_2Enum_2ESUC $(ap$ c_2Earithmetic_2ENUMERAL $V17n)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Enum_2ESUC $V17n))))\wedge(((ap$ c_2Eprim__rec_2EPRE c_2Enum_2E0) $=$
c_2Enum_2E0)$\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap$ c_2Eprim__rec_2EPRE
$(ap$ c_2Earithmetic_2ENUMERAL $V18n)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Eprim__rec_2EPRE $V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$
$(((ap$ c_2Earithmetic_2ENUMERAL $V19n) =$ c_2Enum_2E0)$\Leftrightarrow(V19n =$ c_2Earithmetic_2EZERO)))$\wedge$
$((\forall V20n \in ty\_2Enum\_2Enum.(($c_2Enum_2E0 $= (ap$ c_2Earithmetic_2ENUMERAL
$V20n))\Leftrightarrow(V20n =$ c_2Earithmetic_2EZERO)))$\wedge((\forall V21n \in ty\_2Enum\_2Enum.$
$(\forall V22m \in ty\_2Enum\_2Enum.(((ap$ c_2Earithmetic_2ENUMERAL
$V21n) = (ap$ c_2Earithmetic_2ENUMERAL $V22m))\Leftrightarrow(V21n = V22m))))\wedge$
$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap$ c_2Eprim__rec_2E_3C
$V23n)$ c_2Enum_2E0))$\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Eprim__rec_2E_3C c_2Enum_2E0) $(ap$ c_2Earithmetic_2ENUMERAL
$V24n)))\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C c_2Earithmetic_2EZERO)
$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Eprim__rec_2E_3C $(ap$ c_2Earithmetic_2ENUMERAL
$V25n))\ (ap$ c_2Earithmetic_2ENUMERAL $V26m)))\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C
$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap$ c_2Earithmetic_2E_3E
c_2Enum_2E0) $V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Earithmetic_2E_3E $(ap$ c_2Earithmetic_2ENUMERAL
$V28n))$ c_2Enum_2E0))$\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C c_2Earithmetic_2EZERO)
$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Earithmetic_2E_3E $(ap$ c_2Earithmetic_2ENUMERAL
$V29n))\ (ap$ c_2Earithmetic_2ENUMERAL $V30m)))\Leftrightarrow(p\ (ap\ (ap$ c_2Eprim__rec_2E_3C
$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap$ c_2Earithmetic_2E_3C_3D
c_2Enum_2E0) $V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$

$((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO})$

$V0n)) = V0n) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$

$V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = V0n) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ ($

$ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ ($

$ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ ($

$ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge$

$(((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$

$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$

$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ($

$((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$

$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$

$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ($

$((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$

$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$

$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ($

$((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO})$

$V0n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$

$V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((ap$

$\mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$

$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$

$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ($

$((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$

$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$

$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ($

$((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$

$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$

$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ($

$((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$

$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$

$(ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge$

$(((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO})$

$V0n)) = (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}$

$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = ($

$ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ ($

$ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$

$V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$

$V0n)\ V1m)))) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$

$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) =$

$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$

$V0n)\ V1m)))) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$

$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) =$

$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$

$V0n)\ V1m)))) \wedge ((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$

$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) =$

$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$

$V0n)\ V1m))))))))))))))))))))))))$

(75)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n)) \Leftrightarrow$$
$$True) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ \mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge$$
$$(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (\neg (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m)))))))))))) \tag{76}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{77}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{78}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow$$
$$(((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{79}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow$$
$$((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{80}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{81}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ($$
$$(p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg($$
$$p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee$$
$$((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))))) \tag{82}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ($$
$$(p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee$$
$$(\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))))) \tag{83}$$

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Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow(\\ (p\ V1q)\lor(p\ V2r)))\Leftrightarrow(((p\ V0p)\lor(\neg(p\ V1q)))\land(((p\ V0p)\lor(\neg(p\ V2r)))\land \\ ((p\ V1q)\lor((p\ V2r)\lor(\neg(p\ V0p)))))))))))$$

(84)

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow(\\ (p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\lor(p\ V1q))\land(((p\ V0p)\lor(\neg(p\ V2r)))\land((\\ \neg(p\ V1q))\lor((p\ V2r)\lor(\neg(p\ V0p)))))))))))$$

(85)

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\lor \\ (p\ V1q))\land((\neg(p\ V1q))\lor(\neg(p\ V0p)))))))$$

(86)

Theorem 1

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ \forall V2p \in ty\_2Einteger\_2Eint.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\ V1n)\ V0m))\land(\neg(V2p = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))\Rightarrow \\ ((ap\ (ap\ c\_2Einteger\_2Eint\_\_div\ (ap\ (ap\ c\_2Einteger\_2Eint\_\_exp \\ V2p)\ V0m))\ (ap\ (ap\ c\_2Einteger\_2Eint\_\_exp\ V2p)\ V1n)) = (ap\ (ap\ c\_2Einteger\_2Eint\_\_exp \\ V2p)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ V1n)))))))$$

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