

thm_2Einteger_2EINT_GE_REDUCE (TMF3HiJ6XQQe35ps5zmGauA9P2p2W6j27Th)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod \\ A0\ A1) \end{aligned} \quad (2)$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (3)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (4)$$

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap\ P\ x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1P \in 2.V1P)))$

Definition 5 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E_40\ (ty_2Einteger_2Eint\ V0a)))$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (5)$$

Definition 6 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.(c_2Einteger_2Etint_lt\ (V0T1, V1T2))$

Definition 7 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o(p \Rightarrow p \ Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Definition 10 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint. \lambda V1y \in ty_2Einteger_2Eint.$

Definition 11 We define $c_2Einteger_2Eint_ge$ to be $\lambda V0x \in ty_2Einteger_2Eint. \lambda V1y \in ty_2Einteger_2Eint.$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^\omega) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (8)$$

Definition 13 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_numi\ m\ 0)$

Definition 14 We define $c_2\text{Ebool_2E_3F}$ to be $\lambda A.\lambda 27a:\iota.(\lambda V0P \in (2^{A \rightarrow 27a}).(ap\;V0P\;(ap\;(c_2\text{Emin_2E_40}))$

Definition 15 We define $c_2Eprim_rec_2E\lambda C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Ebool_2E_21\ 2))(\lambda V2t \in$

Definition 17 We define $c_2Earthmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

c_2Enum_2ZERO_REPO $\in \omega$

define c_2Enum_2E0 to be (ap c_2Enum_2EABS_num c_2E

Let c_2 be given. Assume the following.

$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum^{ty_2Enum_2Enum}})^*)$

(10)

Definition 20 We define c -2Earithmetic-2EBIT1 to be $\lambda V0n \in tu\ 2Enum\ 2Enum.(an\ (an\ c\ 2Earithmetic-$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)_{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)} \quad (11)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)} \quad (12)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod ty_2Enum_2Enum)}} \quad (13)$$

Definition 21 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$

Definition 22 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint_eq V0T1)$

Definition 23 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint)^{ty_2Enum_2Enum} \quad (14)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

Theorem 1

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\forall V2num \in ty_2Num. \\
& ((p (ap (ap c_2Einteger_2Eint_ge (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \Leftrightarrow \\
& c_2Enum_2E0)) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Einteger_2Eint_ge (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V0n))) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Einteger_2Eint_ge (ap \\
& c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 V0n)))) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Einteger_2Eint_ge (ap \\
& c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V0n)))) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Einteger_2Eint_ge (ap \\
& c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V0n))) \Leftrightarrow \\
& False) \wedge (((p (ap (ap c_2Einteger_2Eint_ge (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V0n))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Einteger_2Eint_ge \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 V0n)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Einteger_2Eint_ge \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V0n)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Einteger_2Eint_ge \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1m))) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V0n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0n) V1m))) \wedge (((p (\\
& ap (ap c_2Einteger_2Eint_ge (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V1m)))) \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V0n))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Einteger_2Eint_ge (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V1m)))) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V0n))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Einteger_2Eint_ge \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1m))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Einteger_2Eint_ge \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V0n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1m) V0n)))))))))))))))
\end{aligned}$$