thm_2Einteger_2EINT__LE_DOUBLE (TMaWybkaVoE8dKaRm4eNgDYqXBFJsmdCK4v)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\text{-}27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\text{-}27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (2)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (3)$$

Definition 5 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \qquad (4)$$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \qquad (5)$$

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Let $ty\_2Epair\_2Eprod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Epair\_2Eprod$$
$$A0\ A1) \tag{6}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2}$$
$$\tag{7}$$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota{\Rightarrow}\iota.$

Definition 9 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap\ ($c_2Emin_2E_40 $(ty.$

Let $c\_2Einteger\_2Etint\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum}$$
$$\tag{8}$$

Definition 10 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$.

Let $c\_2Einteger\_2Etint\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty}$$
$$\tag{9}$$

Let $c\_2Einteger\_2Etint\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum}$$
$$\tag{10}$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}}$$
$$\tag{11}$$

Definition 11 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 12 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteg$

Definition 13 *We define* c_2Einteger_2Eint__le *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2E$

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Assume the following.
$$True \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1){\Rightarrow}(p\ V1t2)){\Rightarrow}(((p$$
$$V1t2){\Rightarrow}(p\ V0t1)){\Rightarrow}((p\ V0t1){\Leftrightarrow}(p\ V1t2)))))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow$$
$$True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\quad (14)$$
$$(p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t))))))))$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}}$$
$$V0x)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})) = V0x)) \quad (15)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((\neg(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ V0x)\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}}$$
$$V1y)\ V0x)))))$$

$$(16)$$

Assume the following.

$$(\forall V0w \in ty\_2Einteger\_2Eint.(\forall V1x \in ty\_2Einteger\_2Eint.$$
$$(\forall V2y \in ty\_2Einteger\_2Eint.(\forall V3z \in ty\_2Einteger\_2Eint.$$
$$(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ V0w)\ V1x)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}}$$
$$V2y)\ V3z))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}}$$
$$V0w)\ V2y))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1x)\ V3z)))))))))$$

$$(17)$$

Assume the following.

$$(\forall V0w \in ty\_2Einteger\_2Eint.(\forall V1x \in ty\_2Einteger\_2Eint.$$
$$(\forall V2y \in ty\_2Einteger\_2Eint.(\forall V3z \in ty\_2Einteger\_2Eint.$$
$$(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ V0w)\ V1x)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}}$$
$$V2y)\ V3z))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}}$$
$$V0w)\ V2y))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1x)\ V3z)))))))))$$

$$(18)$$

Theorem 1

$$(\forall V0x \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}}$$
$$V0x)\ V0x))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num}$$
$$\mathsf{c\_2Enum\_2E0}))\ V0x))))$$