

# thm\_2Einteger\_2EINT\_\_LE\_\_MUL (TMX- AgVS6wvBCu2gtcQtJwPx25U5kRxNn8eH)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow & nonempty\ (ty\_2Epair\_2Eprod \\ & A0\ A1) \end{aligned} \quad (2)$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (3)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (4)$$

Let  $c\_2Enum\_2EAABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EAABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (5)$$

**Definition 7** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EAABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 8** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (6)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (7)$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num ($

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 10** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic_$

**Definition 11** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (9)$$

**Definition 13** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2$

**Definition 14** We define  $c\_2Einteger\_2Etint\_0$  to be  $(ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum ty\_2Enum ty\_2Enum$

Let  $c\_2Einteger\_2Etint\_mul : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_mul \in (((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)}) \quad (10)$$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Einteger\_2Eint \quad (11)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)}}) \quad (12)$$

**Definition 15** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum ty\_2Enum)$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Einteger\_2Eint)}) \quad (13)$$

**Definition 16** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge p(x)) \text{ else } \perp$ .

**Definition 17** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint. (ap (c\_2Emin\_2E\_40 (ty\_2Einteger\_2Eint)))$

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)} \quad (14)$$

**Definition 18** We define  $c\_2Einteger\_2Eint\_0$  to be  $(ap c\_2Einteger\_2Eint\_ABS c\_2Einteger\_2Etint\_0)$ .

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint)^{ty\_2Enum\_2Enum} \quad (15)$$

**Definition 19** We define  $c\_2Einteger\_2Eint\_mul$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint. \lambda V1T2 \in ty\_2Einteger\_2Eint. (ap (c\_2Einteger\_2Eint\_0 (V0T1 * V1T2)))$

**Definition 20** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint. \lambda V1T2 \in ty\_2Einteger\_2Eint. (ap (c\_2Einteger\_2Eint\_0 (V0T1 < V1T2)))$

**Definition 21** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. (ap (c\_2Ebool\_2E\_21 1) (V2t)))))))$

**Definition 22** We define  $c\_2Einteger\_2Eint\_le$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint. \lambda V1y \in ty\_2Einteger\_2Eint. (ap (c\_2Einteger\_2Eint\_0 (V0x <= V1y)))$

**Definition 23** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x = V1y)))$

**Definition 24** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27b})) . (V0f (A\_27a, A\_27b))$

**Definition 25** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) A\_27b) A\_27c)$

**Definition 26** We define  $c\_2Equotient\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda A\_27d : \iota. \lambda V0f \in ((A\_27c^{A\_27b})^{A\_27b}) . (V0f (A\_27a, A\_27b, A\_27c, A\_27d))$

**Definition 27** We define  $c\_2Equotient\_2E\_3D\_3D\_3D\_3E$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R1 \in ((2^{A\_27a})^{A\_27a}) . (R1 (A\_27a, A\_27b))$

**Definition 28** We define  $c\_2Equotient\_2EQOUTIENT$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}) . (R (A\_27a, A\_27b))$

**Definition 29** We define  $c\_2Ecombin\_2EW$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a})) . (V0f (A\_27a, A\_27b))$

**Definition 30** We define  $c\_2Equotient\_2Erespects$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (c\_2Ecombin\_2EW A\_27a A\_27b) = 1$

**Definition 31** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap V1f V0x)))$

**Definition 32** We define  $c\_2Ebool\_2ERES\_FORALL$  to be  $\lambda A\_27a : \iota. (\lambda V0p \in (2^{A\_27a}). (\lambda V1m \in (2^{A\_27a}). (ap V1m V0p)))$

**Definition 33** We define  $c\_2Equotient\_2EEQUIV$  to be  $\lambda A\_27a : \iota. \lambda V0E \in ((2^{A\_27a})^{A\_27a}). (ap (c\_2Ebool\_2EQOUTIENT A\_27a) V0E)$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (22)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (26)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap (c\_2Ecombin\_2EI A\_27a) V0x) = V0x)) \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum). \\ & (\forall V1q \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum). \\ & ((p (ap (ap c\_2Einteger\_2Etint\_eq V0p) V1q)) \Leftrightarrow ((ap c\_2Einteger\_2Etint\_eq V0p) = (ap c\_2Einteger\_2Etint\_eq V1q)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum). \\ & (\forall V1y \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum). \\ & (((p (ap (ap c\_2Einteger\_2Etint\_lt c\_2Einteger\_2Etint\_0) V0x)) \wedge \\ & (p (ap (ap c\_2Einteger\_2Etint\_lt c\_2Einteger\_2Etint\_0) V1y))) \Rightarrow \\ & (p (ap (ap c\_2Einteger\_2Etint\_lt c\_2Einteger\_2Etint\_0) (ap \\ & (ap c\_2Einteger\_2Etint\_mul V0x) V1y)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0x1 \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum). \\ & (\forall V1x2 \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum). \\ & (\forall V2y1 \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum). \\ & (\forall V3y2 \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum). \\ & (((p (ap (ap c\_2Einteger\_2Etint\_eq V0x1) V1x2)) \wedge (p (ap (ap c\_2Einteger\_2Etint\_eq V2y1) V3y2))) \Rightarrow (p (ap (ap c\_2Einteger\_2Etint\_eq (ap (ap c\_2Einteger\_2Etint\_mul V0x1) V2y1)) (ap (ap c\_2Einteger\_2Etint\_mul V1x2) V3y2))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0x1 \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum). \\ & (\forall V1x2 \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum). \\ & (\forall V2y1 \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum). \\ & (\forall V3y2 \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum). \\ & (((p (ap (ap c\_2Einteger\_2Etint\_eq V0x1) V1x2)) \wedge (p (ap (ap c\_2Einteger\_2Etint\_eq V2y1) V3y2))) \Rightarrow ((p (ap (ap c\_2Einteger\_2Etint\_lt V0x1) V2y1)) \Leftrightarrow \\ & (p (ap (ap c\_2Einteger\_2Etint\_lt V1x2) V3y2))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum) ty\_2Einteger\_2Eint) c\_2Einteger\_2Etint\_eq) \\ & c\_2Einteger\_2Eint\_ABS) c\_2Einteger\_2Eint\_REP)) \end{aligned} \quad (32)$$

Assume the following.

$$(c\_2Einteger\_2Eint\_0 = (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0)) \quad (33)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty\_2Einteger\_2Eint. ((ap\ (ap\ c\_2Einteger\_2Eint\_mul \\ (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0))\ V0x) = (ap\ c\_2Einteger\_2Eint\_of\_num \\ c\_2Enum\_2E0))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty\_2Einteger\_2Eint. ((ap\ (ap\ c\_2Einteger\_2Eint\_mul \\ V0x)\ (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0)) = (ap\ c\_2Einteger\_2Eint\_of\_num \\ c\_2Enum\_2E0))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. \\ ((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_le\ V0x)\ V1y)) \Leftrightarrow ((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt \\ V0x)\ V1y)) \vee (V0x = V1y)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ A\_27a\ A\_27a)\ (c\_2Emin\_2E\_3D\ A\_27a))\ (c\_2Ecombin\_2EI\ A\_27a))\ ( \\ c\_2Ecombin\_2EI\ A\_27a))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\ ((\forall V1x \in A\_27a. (\forall V2y \in A\_27a. ((p\ (ap\ (ap\ V0R\ V1x)\ V2y)) \Leftrightarrow \\ ((ap\ V0R\ V1x) = (ap\ V0R\ V2y)))))) \Leftrightarrow ((\forall V3x \in A\_27a. (p\ (ap\ (ap\ V0R \\ V3x)\ V3x))) \wedge ((\forall V4x \in A\_27a. (\forall V5y \in A\_27a. ((p\ (ap\ ( \\ ap\ V0R\ V4x)\ V5y)) \Rightarrow (p\ (ap\ (ap\ V0R\ V5y)\ V4x)))))) \wedge (\forall V6x \in A\_27a. \\ (\forall V7y \in A\_27a. (\forall V8z \in A\_27a. (((p\ (ap\ (ap\ V0R\ V6x)\ V7y)) \wedge \\ (p\ (ap\ (ap\ V0R\ V7y)\ V8z)))) \Rightarrow (p\ (ap\ (ap\ V0R\ V6x)\ V8z)))))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \forall A_{27c}. \\
& nonempty A_{27c} \Rightarrow \forall A_{27d}.nonempty A_{27d} \Rightarrow (\forall V0R1 \in ( \\
& (2^{A_{27a}})^{A_{27a}}).(\forall V1abs1 \in (A_{27c})^{A_{27a}}).(\forall V2rep1 \in \\
& (A_{27a})^{A_{27c}}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT A_{27a} A_{27c}) \\
& V0R1) V1abs1) V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_{27b}})^{A_{27b}}).(\forall V4abs2 \in \\
& (A_{27d})^{A_{27b}}).(\forall V5rep2 \in (A_{27b})^{A_{27d}}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& A_{27b} A_{27d}) V3R2) V4abs2) V5rep2)) \Rightarrow (p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& (A_{27b})^{A_{27a}}) (A_{27d})^{A_{27c}}) (ap (ap (c_2Equotient_2E_3D_3D_3D_3E \\
& A_{27a} A_{27b}) V0R1) V3R2)) (ap (ap (c_2Equotient_2E_2D_2D_3E A_{27c} \\
& A_{27b} A_{27a} A_{27d}) V2rep1) V4abs2)) (ap (ap (c_2Equotient_2E_2D_2D_3E \\
& A_{27a} A_{27d} A_{27c} A_{27b}) V1abs1) V5rep2))))))))))) \\
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \forall A_{27c}. \\
& nonempty A_{27c} \Rightarrow \forall A_{27d}.nonempty A_{27d} \Rightarrow (\forall V0R1 \in ( \\
& (2^{A_{27a}})^{A_{27a}}).(\forall V1abs1 \in (A_{27c})^{A_{27a}}).(\forall V2rep1 \in \\
& (A_{27a})^{A_{27c}}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT A_{27a} A_{27c}) \\
& V0R1) V1abs1) V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_{27b}})^{A_{27b}}).(\forall V4abs2 \in \\
& (A_{27d})^{A_{27b}}).(\forall V5rep2 \in (A_{27b})^{A_{27d}}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& A_{27b} A_{27d}) V3R2) V4abs2) V5rep2)) \Rightarrow (\forall V6f \in (A_{27d})^{A_{27c}}). \\
& ((\lambda V7x \in A_{27c}.(ap V6f V7x)) = (ap (ap (ap (c_2Equotient_2E_2D_2D_3E \\
& A_{27c} A_{27b} A_{27a} A_{27d}) V2rep1) V4abs2) (\lambda V8x \in A_{27a}.(ap V5rep2 \\
& (ap V6f (ap V1abs1 V8x))))))))))))))) \\
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow ( \\
& \forall V0REL \in ((2^{A_{27a}})^{A_{27a}}).(\forall V1abs \in (A_{27b})^{A_{27a}}). \\
& (\forall V2rep \in (A_{27a})^{A_{27b}}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& A_{27a} A_{27b}) V0REL) V1abs) V2rep)) \Rightarrow (\forall V3x1 \in A_{27a}.(\forall V4x2 \in \\
& A_{27a}.((p (ap (ap V0REL V3x1) V4x2)) \Rightarrow (p (ap (ap V0REL V3x1) (ap V2rep \\
& (ap V1abs V4x2))))))))))) \\
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow ( \\
& \forall V0R \in ((2^{A_{27a}})^{A_{27a}}).(\forall V1abs \in (A_{27b})^{A_{27a}}). \\
& (\forall V2rep \in (A_{27a})^{A_{27b}}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& A_{27a} A_{27b}) V0R) V1abs) V2rep)) \Rightarrow (\forall V3f \in (2^{A_{27b}}).((p ( \\
& ap (c_2Ebool_2E_21 A_{27b}) V3f)) \Leftrightarrow (p (ap (ap (c_2Ebool_2ERES_FORALL \\
& A_{27a}) (ap (c_2Equotient_2Erespects A_{27a} 2) V0R)) (ap (ap ( \\
& (c_2Equotient_2E_2D_2D_3E A_{27a} 2 A_{27b} 2) V1abs) (c_2Ecombin_2EI \\
& 2)) V3f))))))) \\
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \\
& \quad (\forall V0R \in ((2^{A_{.27a}})^{A_{.27a}}).(\forall V1abs \in (A_{.27b})^{A_{.27a}}). \\
& \quad (\forall V2rep \in (A_{.27a})^{A_{.27b}}).((p (ap (ap (ap (c_{.2Equotient_2EQUOTIENT} \\
& \quad A_{.27a} A_{.27b}) V0R) V1abs) V2rep)) \Rightarrow (\forall V3f \in (2^{A_{.27a}}).(\forall V4g \in \\
& \quad (2^{A_{.27a}}).((p (ap (ap (ap (c_{.2Equotient_2E.3D_3D_3E} A_{.27a} \\
& \quad 2) V0R) (c_{.2Emin_2E_3D} 2)) V3f) V4g)) \Rightarrow ((p (ap (ap (c_{.2Ebool_2ERES_FORALL} \\
& \quad A_{.27a}) (ap (c_{.2Equotient_2Erespects} A_{.27a} 2) V0R)) V3f)) \Leftrightarrow (p ( \\
& \quad ap (ap (c_{.2Ebool_2ERES_FORALL} A_{.27a}) (ap (c_{.2Equotient_2Erespects} \\
& \quad A_{.27a} 2) V0R)) V4g))))))) \\
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}. \\
& \quad nonempty A_{.27c} \Rightarrow \forall A_{.27d}.nonempty A_{.27d} \Rightarrow (\forall V0R1 \in ( \\
& \quad (2^{A_{.27a}})^{A_{.27a}}).(\forall V1abs1 \in (A_{.27c})^{A_{.27a}}).(\forall V2rep1 \in \\
& \quad (A_{.27a})^{A_{.27c}}).((p (ap (ap (c_{.2Equotient_2EQUOTIENT} A_{.27a} A_{.27c}) \\
& \quad V0R1) V1abs1) V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_{.27b}})^{A_{.27b}}).(\forall V4abs2 \in \\
& \quad (A_{.27d})^{A_{.27b}}).(\forall V5rep2 \in (A_{.27b})^{A_{.27d}}).((p (ap (ap (c_{.2Equotient_2EQUOTIENT} \\
& \quad A_{.27b} A_{.27d}) V3R2) V4abs2) V5rep2)) \Rightarrow (\forall V6f \in (A_{.27b})^{A_{.27a}}). \\
& \quad (\forall V7g \in (A_{.27b})^{A_{.27a}}).(\forall V8x \in A_{.27a}.(\forall V9y \in \\
& \quad A_{.27a}).(((p (ap (ap (ap (c_{.2Equotient_2E_3D_3D_3E} A_{.27a} \\
& \quad A_{.27b}) V0R1) V3R2) V6f) V7g)) \wedge (p (ap (ap V0R1 V8x) V9y))) \Rightarrow (p (ap ( \\
& \quad ap V3R2 (ap V6f V8x)) (ap V7g V9y))))))))))) \\
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0E \in ((2^{A_{.27a}})^{A_{.27a}}). \\
& \quad (\forall V1P \in (2^{A_{.27a}}).((p (ap (c_{.2Equotient_2EEQUIV} A_{.27a}) \\
& \quad V0E)) \Rightarrow ((p (ap (ap (c_{.2Ebool_2ERES_FORALL} A_{.27a}) (ap (c_{.2Equotient_2Erespects} \\
& \quad A_{.27a} 2) V0E)) V1P)) \Leftrightarrow (p (ap (c_{.2Ebool_2E.21} A_{.27a}) V1P)))))) \\
\end{aligned} \tag{45}$$

### Theorem 1

$$\begin{aligned}
& (\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint. \\
& \quad (((p (ap (ap c_{.2Einteger_2Eint_le} (ap c_{.2Einteger_2Eint_of_num} \\
& \quad c_{.2Enum_2E0})) V0x)) \wedge (p (ap (ap c_{.2Einteger_2Eint_le} (ap c_{.2Einteger_2Eint_of_num} \\
& \quad c_{.2Enum_2E0})) V1y))) \Rightarrow ((p (ap (ap c_{.2Einteger_2Eint_le} (ap c_{.2Einteger_2Eint_of_num} \\
& \quad c_{.2Enum_2E0})) (ap (ap c_{.2Einteger_2Eint_mul} V0x) V1y)))))))
\end{aligned}$$