

thm\_2Einteger\_2EINT\_\_LTE\_\_ADD  
(TMQ8QbN8KXWUNMDU7mQNBjZdqndKSGeezrU)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 7** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{4}$$

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \tag{5}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (6)$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint\_REP\_CLASS}) \quad (7)$$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p\ (ap\ P\ x))$  **then** (the  $(\lambda x.x \in A \wedge p\ x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 9** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint$ .  $(ap\ (c\_2Emin\_2E\_40\ (ty\_2Einteger\_2Eint\_REP\ a)))$

Let  $c\_2Einteger\_2Eint\_add : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{ty\_2Einteger\_2Eint\_add})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (8)$$

Let  $c\_2Einteger\_2Eint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (9)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})} \quad (10)$$

**Definition 10** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$

**Definition 11** We define  $c\_2Einteger\_2Eint\_add$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint$ .  $\lambda V1T2 \in ty\_2Einteger\_2Eint$

Let  $c\_2Einteger\_2Eint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (11)$$

**Definition 12** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint$ .  $\lambda V1T2 \in ty\_2Einteger\_2Eint$

**Definition 13** We define  $c\_2Einteger\_2Eint\_le$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint$ .  $\lambda V1y \in ty\_2Einteger\_2Eint$

**Definition 14** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (13) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& p \ V0t))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Einteger\_2Eint.((ap \ (ap \ c\_2Einteger\_2Eint\_add \\
& (ap \ c\_2Einteger\_2Eint\_of\_num \ c\_2Enum\_2E0)) \ V0x) = V0x))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0w \in ty\_2Einteger\_2Eint.(\forall V1x \in ty\_2Einteger\_2Eint. \\
& (\forall V2y \in ty\_2Einteger\_2Eint.(\forall V3z \in ty\_2Einteger\_2Eint. \\
& (((p \ (ap \ (ap \ c\_2Einteger\_2Eint\_lt \ V0w) \ V1x)) \wedge (p \ (ap \ (ap \ c\_2Einteger\_2Eint\_le \\
& V2y) \ V3z)))) \Rightarrow (p \ (ap \ (ap \ c\_2Einteger\_2Eint\_lt \ (ap \ (ap \ c\_2Einteger\_2Eint\_add \\
& V0w) \ V2y)) \ (ap \ (ap \ c\_2Einteger\_2Eint\_add \ V1x) \ V3z))))))
\end{aligned} \tag{16}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint. \\
& (((p \ (ap \ (ap \ c\_2Einteger\_2Eint\_lt \ (ap \ c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0)) \ V0x)) \wedge (p \ (ap \ (ap \ c\_2Einteger\_2Eint\_le \ (ap \ c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0)) \ V1y)))) \Rightarrow (p \ (ap \ (ap \ c\_2Einteger\_2Eint\_lt \ (ap \ c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0)) \ (ap \ (ap \ c\_2Einteger\_2Eint\_add \ V0x) \ V1y))))))
\end{aligned}$$