

thm_2Einteger_2EINT__LTE__ADD2 (TMdzQWrvLzcd2uPkrJ4kB1HjUUNoyy8Hka)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V 0t \in 2. V 0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V 0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V 0t)) (\text{c_2Ebool_2E_2F } V 0t)))$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \tag{1}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A 0 A 1) \tag{2}$$

Let `c_2Einteger_2Etint_eq` : ι be given. Assume the following.

$$\text{c_2Einteger_2Etint_eq} \in ((2^{(\text{ty_2Epair_2Eprod } \text{ty_2Enum_2Enum } \text{ty_2Enum_2Enum})}) (\text{ty_2Epair_2Eprod } \text{ty_2Enum_2Enum})) \tag{3}$$

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Einteger_2Eint} \tag{4}$$

Let `c_2Einteger_2Eint__REP__CLASS` : ι be given. Assume the following.

$$\text{c_2Einteger_2Eint_REP_CLASS} \in ((2^{(\text{ty_2Epair_2Eprod } \text{ty_2Enum_2Enum } \text{ty_2Enum_2Enum})}) (\text{ty_2Einteger_2Eint})) \tag{5}$$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x.x \in A \backslash p \text{ of type } \iota \Rightarrow \iota).$

Definition 8 We define `c_2Einteger_2Eint__REP` to be $\lambda V0a \in \text{ty_2Einteger_2Eint. (ap (c_2Emin_2E_40 (ty_2Einteger_2Eint_add : \iota$

$$c_2Einteger_2Eint_add \in (((\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum})^{\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}})^{\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}})^{\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}} \quad (6)$$

Let `c_2Einteger_2Eint__ABS__CLASS` : ι be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (\text{ty_2Einteger_2Eint})^{2^{(\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum})}} \quad (7)$$

Definition 9 We define `c_2Einteger_2Eint__ABS` to be $\lambda V0r \in (\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum})$

Definition 10 We define `c_2Einteger_2Eint__add` to be $\lambda V0T1 \in \text{ty_2Einteger_2Eint.} \lambda V1T2 \in \text{ty_2Einteger_2Eint.}$

Let `c_2Einteger_2Eint__lt` : ι be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum})})^{\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}})^{\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}} \quad (8)$$

Definition 11 We define `c_2Einteger_2Eint__lt` to be $\lambda V0T1 \in \text{ty_2Einteger_2Eint.} \lambda V1T2 \in \text{ty_2Einteger_2Eint.}$

Definition 12 We define `c_2Einteger_2Eint__le` to be $\lambda V0x \in \text{ty_2Einteger_2Eint.} \lambda V1y \in \text{ty_2Einteger_2Eint.}$

Definition 13 We define `c_2Ecombin_2EK` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 14 We define `c_2Ecombin_2ES` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 15 We define `c_2Ecombin_2EI` to be $\lambda A_27a : \iota. (\text{ap (ap (c_2Ecombin_2ES } A_27a \text{ (} A_27a^{A_27a})) A_27a))$

Definition 16 We define `c_2Equotient_2E_2D_2D_3E` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda A_27d : \iota. \lambda V0f \in ((A_27d^{A_27b})^{A_27a})$

Definition 17 We define `c_2Equotient_2E_3D_3D_3E` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R1 \in ((2^{A_27a})^{A_27b})$

Definition 18 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap (c_2Ebool_2E_21 } 2) \ (\lambda V2t \in 2. \text{ap (c_2Ebool_2E_21 } 2) \ t1))))$

Definition 19 We define `c_2Equotient_2EQUOTIENT` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27b}).$

Definition 20 We define `c_2Ecombin_2EW` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in ((A_27b^{A_27a})^{A_27a}). (\lambda V1x \in A_27a. \text{ap (c_2Ecombin_2EW } A_27a \text{ } A_27b) \ x)))$

Definition 21 We define `c_2Equotient_2ERespects` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\text{c_2Ecombin_2EW } A_27a \text{ } A_27b)$

Definition 22 We define `c_2Ebool_2EIN` to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (\text{ap } V1f \ V0x)))$

Definition 23 We define `c_2Ebool_2ERES__FORALL` to be $\lambda A_27a : \iota. (\lambda V0p \in (2^{A_27a}). (\lambda V1m \in (2^{A_27a}). (\text{ap } V1m \ V0p)))$

Definition 24 We define $c_2Equotient_2EEQUIV$ to be $\lambda A_27a : \iota.\lambda V0E \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \wedge (p V1t2)) \Leftrightarrow ((p V1t2) \wedge (p V0t1)))))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \tag{11}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \tag{13}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Ecombin_2EI A_27a) V0x) = V0x)) \tag{14}$$

Assume the following.

$$(\forall V0p \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). (\forall V1q \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). ((p (ap (ap c_2Einteger_2Etint_eq V0p) V1q)) \Leftrightarrow ((ap c_2Einteger_2Etint_eq V0p) = (ap c_2Einteger_2Etint_eq V1q)))))) \tag{15}$$

Assume the following.

$$(\forall V0p \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). (\forall V1q \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). ((V0p = V1q) \Rightarrow (p (ap (ap c_2Einteger_2Etint_eq V0p) V1q)))))) \tag{16}$$

Assume the following.

$$(\forall V0x \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). (\forall V1y \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). ((ap (ap c_2Einteger_2Etint_add V0x) V1y) = (ap (ap c_2Einteger_2Etint_add V1y) V0x)))) \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1x2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V2y1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V3y2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (((p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V2y1)\ V3y2))) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ (ap\ (ap\ c_2Einteger_2Etint_add\ V0x1)\ V2y1))\ (ap\ (ap\ c_2Einteger_2Etint_add\ V1x2)\ V3y2))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)\ ty_2Einteger_2Eint)\ c_2Einteger_2Etint_eq)\ c_2Einteger_2Eint_ABS)\ c_2Einteger_2Eint_REP))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0w \in ty_2Einteger_2Eint. (\forall V1x \in ty_2Einteger_2Eint. \\
& (\forall V2y \in ty_2Einteger_2Eint. (\forall V3z \in ty_2Einteger_2Eint. \\
& (((p\ (ap\ (ap\ c_2Einteger_2Eint_le\ V0w)\ V1x)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ V2y)\ V3z))) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ (ap\ c_2Einteger_2Eint_add\ V0w)\ V2y))\ (ap\ (ap\ c_2Einteger_2Eint_add\ V1x)\ V3z))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27a)\ (c_2Emin_2E_3D\ A_27a))\ (c_2Ecombin_2EI\ A_27a))\ (c_2Ecombin_2EI\ A_27a)))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& (2^{A_27a})^{A_27a}). (\forall V1abs1 \in (A_27c)^{A_27a}). (\forall V2rep1 \in \\
& (A_27a)^{A_27c}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c)\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}). (\forall V4abs2 \in \\
& (A_27d)^{A_27b}). (\forall V5rep2 \in (A_27b)^{A_27d}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27b\ A_27a)\ (A_27b)^{A_27a})\ (A_27d)^{A_27c})\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a\ A_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27a\ A_27d\ A_27c\ A_27b)\ V1abs1)\ V5rep2))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x \in A_27b.(\forall V4y \in \\
& A_27b.((V3x = V4y) \Leftrightarrow (p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3x))\ (ap\ V2rep\ V4y)))))))))) \\
& \hspace{15em} (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& A_27a.(\forall V5y1 \in A_27a.(\forall V6y2 \in A_27a.(((p\ (ap\ (ap\ V0R \\
& V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2)))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2)))))))))) \\
& \hspace{15em} (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (\forall V6f \in (A_27d^{A_27c}). \\
& ((\lambda V7x \in A_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x)))))))))) \\
& \hspace{15em} (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0REL \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0REL)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& A_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2)))))))))) \\
& \hspace{15em} (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27b}).((p\ (\\
& ap\ (c_2Ebool_2E_21\ A_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b\ 2)\ V1abs)\ (c_2Ecombin_2EI \\
& \quad 2))\ V3f))))))))) \\
& \hspace{15em} (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27a}).(\forall V4g \in \\
& \quad (2^{A_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad 2)\ V0R)\ (c_2Emin_2E_3D\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (\\
& ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0R))\ V4g))))))))) \\
& \hspace{15em} (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b^{A_27a}). \\
& \quad (\forall V7g \in (A_27b^{A_27a}).(\forall V8x \in A_27a.(\forall V9y \in \\
& \quad A_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))) \\
& \hspace{15em} (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0E \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1P \in (2^{A_27a}).((p\ (ap\ (c_2Equotient_2EEQUIV\ A_27a) \\
& V0E)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E_21\ A_27a)\ V1P)))))) \\
& \hspace{15em} (30)
\end{aligned}$$

Theorem 1

$$\begin{aligned} & (\forall V0w \in ty_2Einteger_2Eint. (\forall V1x \in ty_2Einteger_2Eint. \\ & (\forall V2y \in ty_2Einteger_2Eint. (\forall V3z \in ty_2Einteger_2Eint. \\ & (((p (ap (ap c_2Einteger_2Eint_lt V0w) V1x)) \wedge (p (ap (ap c_2Einteger_2Eint_le \\ & V2y) V3z)))) \Rightarrow (p (ap (ap c_2Einteger_2Eint_lt (ap (ap c_2Einteger_2Eint_add \\ & V0w) V2y)) (ap (ap c_2Einteger_2Eint_add V1x) V3z)))))))) \end{aligned}$$