thm_2Einteger_2EINT__LTE__TOTAL (TMKtX-tQzoNrCXChHB6H7GU9cyVJaqmT1Tkz)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P {\Rightarrow} p\ Q)$
of type ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota {\Rightarrow} \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\textsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (\textsf{c\_2Emin\_2E\_3D}\ (2^{A-27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\textsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\textsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\textsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota {\Rightarrow} \iota {\Rightarrow} \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 {\Rightarrow} \forall A1.nonempty\ A1 {\Rightarrow} nonempty\ (ty\_2Epair\_2Eprod \\ A0\ A1)$$
$$\tag{2}$$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{3}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2}$$
$$\tag{4}$$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A {\wedge} p$
of type $\iota {\Rightarrow} \iota$.

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Definition 9 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 *(ty.*

Let $c\_2Einteger\_2Etint\_\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum.} \tag{5}$$

Definition 10 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger.$

Definition 11 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 12 *We define* c_2Einteger_2Eint__le *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2E$

Assume the following.
$$True \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \tag{7}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge(((p\ V0t){\Leftrightarrow}True){\Leftrightarrow}$$
$$(p\ V0t))\wedge(((False{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(\neg(p\ V0t)))\wedge(((p\ V0t){\Leftrightarrow}False){\Leftrightarrow}(\neg(\tag{8}$$
$$p\ V0t)))))))$$

Theorem 1

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ V0x)\ V1y))\vee(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}$$
$$V1y)\ V0x)))))$$