

thm_2Einteger_2EINT__LT__MUL2 (TMXJW26ZHaZiYTLMe5BNPnfbuoVjYN6zNdP)

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Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2E2T to be $(ap (ap (c_2Emin_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2E21 to be $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E3D (2^{A-27a}))$

Definition 4 We define c_2Ebool_2E2F to be $(ap (c_2Ebool_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E3D_3D_3E V0t) c_2Ebool_2E2F$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})(ty_2Epair_2Eprod\ ty_2Enum_2Enum)) \tag{3}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{5}$$

Definition 7 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 8 We define $c_2\text{Earithmetic_EZERO}$ to be $c_2\text{Enum_E0}$.

Let $c_2\text{Enum_EREP_num} : \iota$ be given. Assume the following.

$$c_2\text{Enum_EREP_num} \in (\text{omega}^{ty_2\text{Enum_Enum}}) \quad (6)$$

Let $c_2\text{Enum_ESUC_REP} : \iota$ be given. Assume the following.

$$c_2\text{Enum_ESUC_REP} \in (\text{omega}^{\text{omega}}) \quad (7)$$

Definition 9 We define $c_2\text{Enum_ESUC}$ to be $\lambda V0m \in ty_2\text{Enum_Enum} . (ap\ c_2\text{Enum_EABS_num})$

Let $c_2\text{Earithmetic_E_2B} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_E_2B} \in ((ty_2\text{Enum_Enum}^{ty_2\text{Enum_Enum}})^{ty_2\text{Enum_Enum}}) \quad (8)$$

Definition 10 We define $c_2\text{Earithmetic_EBIT1}$ to be $\lambda V0n \in ty_2\text{Enum_Enum} . (ap\ (ap\ c_2\text{Earithmetic_E_2B}))$

Definition 11 We define $c_2\text{Earithmetic_ENUMERAL}$ to be $\lambda V0x \in ty_2\text{Enum_Enum} . V0x$.

Definition 12 We define $c_2\text{Ebool_E_2F_5C}$ to be $(\lambda V0t1 \in 2 . (\lambda V1t2 \in 2 . (ap\ (c_2\text{Ebool_E_21}\ 2))\ (\lambda V2t \in 2 . V0t))))$

Let $c_2\text{Epair_EABS_prod} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a . \text{nonempty } A_27a \Rightarrow \forall A_27b . \text{nonempty } A_27b \Rightarrow c_2\text{Epair_EABS_prod} \\ A_27a\ A_27b \in ((ty_2\text{Epair_Eprod } A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (9)$$

Definition 13 We define $c_2\text{Epair_E_2C}$ to be $\lambda A_27a : \iota . \lambda A_27b : \iota . \lambda V0x \in A_27a . \lambda V1y \in A_27b . (ap\ (c_2\text{Epair_EABS_prod}))\ x\ y$

Definition 14 We define $c_2\text{Einteger_Eint_0}$ to be $(ap\ (ap\ (c_2\text{Epair_E_2C}\ ty_2\text{Enum_Enum}\ ty_2\text{Enum_Enum}))\ 0)$

Let $c_2\text{Einteger_Eint_add} : \iota$ be given. Assume the following.

$$c_2\text{Einteger_Eint_add} \in (((ty_2\text{Epair_Eprod } ty_2\text{Enum_Enum}\ ty_2\text{Enum_Enum})^{(ty_2\text{Epair_Eprod } ty_2\text{Enum_Enum}\ ty_2\text{Enum_Enum})})^{(ty_2\text{Epair_Eprod } ty_2\text{Enum_Enum}\ ty_2\text{Enum_Enum})}) \quad (10)$$

Let $c_2\text{Einteger_Eint_mul} : \iota$ be given. Assume the following.

$$c_2\text{Einteger_Eint_mul} \in (((ty_2\text{Epair_Eprod } ty_2\text{Enum_Enum}\ ty_2\text{Enum_Enum})^{(ty_2\text{Epair_Eprod } ty_2\text{Enum_Enum}\ ty_2\text{Enum_Enum})})^{(ty_2\text{Epair_Eprod } ty_2\text{Enum_Enum}\ ty_2\text{Enum_Enum})}) \quad (11)$$

Let $ty_2\text{Einteger_Eint} : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2\text{Einteger_Eint} \quad (12)$$

Let $c_2\text{Einteger_Eint_ABS_CLASS} : \iota$ be given. Assume the following.

$$c_2\text{Einteger_Eint_ABS_CLASS} \in (ty_2\text{Einteger_Eint})^{(2^{(ty_2\text{Epair_Eprod } ty_2\text{Enum_Enum}\ ty_2\text{Enum_Enum})})} \quad (13)$$

Definition 15 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$.
Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint_REP_CLASS})$$
 (14)

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge P\ x)$) of type $\iota \Rightarrow \iota$.

Definition 17 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E_40\ ty_2Enum_2Enum))$.
Let $c_2Einteger_2Eint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})$$
 (15)

Definition 18 We define $c_2Einteger_2Eint_0$ to be $(ap\ c_2Einteger_2Eint_ABS\ c_2Einteger_2Eint_0)$.

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Einteger_2Eint_neg})$$
 (16)

Definition 19 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint_neg)$.

Definition 20 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.$

Definition 21 We define $c_2Einteger_2Eint_sub$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint.$

Definition 22 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.$

Definition 23 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint.$

Definition 24 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint.$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum})$$
 (17)

Definition 25 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.(\lambda V0x \in A.\lambda V1y \in A.\lambda V2z \in A.$

Definition 26 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A.\lambda V1x \in A.\lambda V2y \in A.$

Definition 27 We define $c_2Ecombin_2EI$ to be $\lambda A.\lambda a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A.\lambda V1x \in A.\lambda V2y \in A.$

Definition 28 We define $c_2Equotient_2E_2D_2D_3E$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.\lambda A.\lambda d : \iota.\lambda V0f \in ((A.\lambda V1x \in A.$

Definition 29 We define $c_2Equotient_2E_3D_3D_3D_3E$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda V0R1 \in ((2^{A.\lambda V1x \in A.$

Definition 30 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda$

Definition 31 We define $c_2Ecombin_2EW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in ((A_27b^{A_27a})^{A_27a}).(\lambda V1x \in$

Definition 32 We define $c_2Equotient_2ERespects$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(c_2Ecombin_2EW A_27a A_27b$

Definition 33 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(\lambda p V1f V0x) \in$

Definition 34 We define $c_2Ebool_2ERES_FORALL$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).(\lambda$

Definition 35 We define $c_2Equotient_2EEQUIV$ to be $\lambda A_27a : \iota.\lambda V0E \in ((2^{A_27a})^{A_27a}).(\lambda p (c_2Ebool_2EIN$

Assume the following.

$$True \tag{18}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t)))))) \end{aligned} \tag{20}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{21}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{22}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg (p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg (\\ & p V0t)))))) \end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \tag{24}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((\lambda p (c_2Ecombin_2EI A_27a) V0x) = V0x)) \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1q \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& ((p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0p)\ V1q)) \Leftrightarrow ((ap\ c_2Einteger_2Etint_eq \\
& \quad V0p) = (ap\ c_2Einteger_2Etint_eq\ V1q))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1q \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \tag{27} \\
& ((V0p = V1q) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0p)\ V1q))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1y \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& ((ap\ (ap\ c_2Einteger_2Etint_add\ V0x)\ V1y) = (ap\ (ap\ c_2Einteger_2Etint_add \\
& \quad V1y)\ V0x))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1y \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V2z \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& ((ap\ (ap\ c_2Einteger_2Etint_add\ V0x)\ (ap\ (ap\ c_2Einteger_2Etint_add \\
& \quad V1y)\ V2z)) = (ap\ (ap\ c_2Einteger_2Etint_add\ (ap\ (ap\ c_2Einteger_2Etint_add \\
& \quad V0x)\ V1y))\ V2z))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1y \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (((p\ (ap\ (ap\ c_2Einteger_2Etint_lt\ c_2Einteger_2Etint_0)\ V0x)) \wedge \\
& \quad (p\ (ap\ (ap\ c_2Einteger_2Etint_lt\ c_2Einteger_2Etint_0)\ V1y))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ c_2Einteger_2Etint_lt\ c_2Einteger_2Etint_0)\ (ap \\
& \quad \quad (ap\ c_2Einteger_2Etint_mul\ V0x)\ V1y))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1x2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V2y1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V3y2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (((p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Etint_eq \\
& \quad V2y1)\ V3y2))) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ (ap\ (ap\ c_2Einteger_2Etint_add \\
& \quad \quad V0x1)\ V2y1))\ (ap\ (ap\ c_2Einteger_2Etint_add\ V1x2)\ V3y2))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1x2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V2y1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V3y2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (((p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V2y1)\ V3y2))) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ (ap\ (ap\ c_2Einteger_2Etint_mul\ V0x1)\ V2y1))\ (ap\ (ap\ c_2Einteger_2Etint_mul\ V1x2)\ V3y2))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1x2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V2y1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V3y2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (((p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V2y1)\ V3y2))) \Rightarrow ((p\ (ap\ (ap\ c_2Einteger_2Etint_lt\ V0x1)\ V2y1)) \Leftrightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_lt\ V1x2)\ V3y2))))))
\end{aligned} \tag{33}$$

Assume the following.

$$(p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)\ ty_2Einteger_2Eint)\ c_2Einteger_2Etint_eq)\ c_2Einteger_2Eint_ABS)\ c_2Einteger_2Eint_REP)) \tag{34}$$

Assume the following.

$$(c_2Einteger_2Eint_0 = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) \tag{35}$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint. ((ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))\ V0x) = V0x)) \tag{36}$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint. ((ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ c_2Einteger_2Eint_neg\ V0x))\ V0x) = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))) \tag{37}$$

Assume the following.

$$((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ V0x)\ V1y)) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Eint_le\ V0x)\ V1y)))) \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. (((p (ap (ap c_2Einteger_2Eint_le \\
V0x) V1y)) \wedge (p (ap (ap c_2Einteger_2Eint_lt V1y) V2z))) \Rightarrow (p (ap \\
& (ap c_2Einteger_2Eint_lt V0x) V2z))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (((p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num \\
c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num \\
c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num \\
c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_mul V0x) V1y))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_sub V0x) V1y))) \Leftrightarrow (p (\\
& ap (ap c_2Einteger_2Eint_lt V1y) V0x))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
V0x) (ap (ap c_2Einteger_2Eint_sub V1y) V2z)) = (ap (ap c_2Einteger_2Eint_sub \\
& (ap (ap c_2Einteger_2Eint_mul V0x) V1y)) (ap (ap c_2Einteger_2Eint_mul \\
V0x) V2z))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
(ap (ap c_2Einteger_2Eint_sub V0x) V1y)) V2z) = (ap (ap c_2Einteger_2Eint_sub \\
& (ap (ap c_2Einteger_2Eint_mul V0x) V2z)) (ap (ap c_2Einteger_2Eint_mul \\
V1y) V2z))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num \\
c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add V0x) V1y))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27a)\ (c_2Emin_2E_3D\ A_27a)\ (c_2Ecombin_2EI\ A_27a)\ (c_2Ecombin_2EI\ A_27a)))))) \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\ & ((\forall V1x \in A_27a. (\forall V2y \in A_27a. ((p\ (ap\ (ap\ V0R\ V1x)\ V2y))) \Leftrightarrow \\ & ((ap\ V0R\ V1x) = (ap\ V0R\ V2y)))))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ (ap\ V0R \\ & V3x)\ V3x))) \wedge ((\forall V4x \in A_27a. (\forall V5y \in A_27a. ((p\ (ap\ (\\ & ap\ V0R\ V4x)\ V5y))) \Rightarrow (p\ (ap\ (ap\ V0R\ V5y)\ V4x)))))) \wedge (\forall V6x \in A_27a. \\ & (\forall V7y \in A_27a. (\forall V8z \in A_27a. (((p\ (ap\ (ap\ V0R\ V6x)\ V7y)) \wedge \\ & (p\ (ap\ (ap\ V0R\ V7y)\ V8z))) \Rightarrow (p\ (ap\ (ap\ V0R\ V6x)\ V8z)))))))))) \quad (46) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\ & (2^{A_27a})^{A_27a}). (\forall V1abs1 \in (A_27c^{A_27a}). (\forall V2rep1 \in \\ & (A_27a^{A_27c}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\ & V0R1)\ V1abs1)\ V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}). (\forall V4abs2 \in \\ & (A_27d^{A_27b}). (\forall V5rep2 \in (A_27b^{A_27d}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & (A_27b^{A_27a})\ (A_27d^{A_27c})\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\ & A_27a\ A_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27c \\ & A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\ & A_27a\ A_27d\ A_27c\ A_27b)\ V1abs1)\ V5rep2)))))))))) \quad (47) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b^{A_27a}). \\ & (\forall V2rep \in (A_27a^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x \in A_27b. (\forall V4y \in \\ & A_27b. ((V3x = V4y) \Leftrightarrow (p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3x))\ (ap\ V2rep\ V4y)))))))))) \quad (48) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0R \in ((2^{A_27a})^{A_27a}). (\forall V1abs \in (A_27b^{A_27a}). \\ & (\forall V2rep \in (A_27a^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x1 \in A_27a. (\forall V4x2 \in \\ & A_27a. (\forall V5y1 \in A_27a. (\forall V6y2 \in A_27a. (((p\ (ap\ (ap\ V0R \\ & V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow \\ & (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2)))))))))) \quad (49) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27d^{A_27c}). \\
& \quad ((\lambda V7x \in A_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x))))))))))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0REL \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& \quad A_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2))))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27b}).((p\ (\\
& \quad ap\ (c_2Ebool_2E_21\ A_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b\ 2)\ V1abs)\ (c_2Ecombin_2EI \\
& \quad 2))\ V3f))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27a}).(\forall V4g \in \\
& \quad (2^{A_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad 2)\ V0R)\ (c_2Emin_2E_3D\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (\\
& \quad ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0R))\ V4g))))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b^{A_27a}). \\
& \quad (\forall V7g \in (A_27b^{A_27a}).(\forall V8x \in A_27a.(\forall V9y \in \\
& \quad A_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y)))))))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0E \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1P \in (2^{A_27a}).((p\ (ap\ (c_2Equotient_2EEQUIV\ A_27a) \\
& \quad V0E)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E.21\ A_27a)\ V1P))))))
\end{aligned} \tag{55}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x1 \in ty_2Einteger_2Eint.(\forall V1x2 \in ty_2Einteger_2Eint. \\
& \quad (\forall V2y1 \in ty_2Einteger_2Eint.(\forall V3y2 \in ty_2Einteger_2Eint. \\
& \quad (((p\ (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ c_2Einteger_2Eint_of_num \\
& \quad c_2Enum_2E0))\ V0x1)) \wedge ((p\ (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ c_2Einteger_2Eint_of_num \\
& \quad c_2Enum_2E0))\ V2y1)) \wedge ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ V0x1) \\
& \quad V1x2)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ V2y1)\ V3y2)))))) \Rightarrow (p\ (ap \\
& \quad (ap\ c_2Einteger_2Eint_lt\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x1) \\
& \quad V2y1))\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V1x2)\ V3y2))))))
\end{aligned}$$