

thm\_2Einteger\_2EINT\_\_LT\_\_NEG  
(TMTQ4QnZAwPeHV8yvvcvEcc5sGtfcU6pCLRe)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2T` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

**Definition 4** We define `c_2Ebool_2E_2F` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let `c_2Einteger_2E_2tint_2Eq` :  $\iota$  be given. Assume the following.

$$c\_2Einteger\_2E\_2tint\_2Eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)) \tag{3}$$

Let `ty_2Einteger_2E_2int` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2E\_2int \tag{4}$$

Let `c_2Einteger_2E_2int_2REP_2CLASS` :  $\iota$  be given. Assume the following.

$$c\_2Einteger\_2E\_2int\_2REP\_2CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) ty\_2Einteger\_2E\_2int) \tag{5}$$

**Definition 7** We define  $c\_2Emin\_2E.40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 8** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E.40 (ty$

Let  $c\_2Einteger\_2Eint\_add : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_add \in (((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (6)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})} \quad (7)$$

**Definition 9** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (8)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (9)$$

**Definition 10** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint)^{ty\_2Enum\_2Enum} \quad (10)$$

Let  $c\_2Einteger\_2Eint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_neg \in (((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (11)$$

**Definition 11** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap c\_2Einteger\_2Eint$

**Definition 12** We define  $c\_2Einteger\_2Eint\_add$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Let  $c\_2Einteger\_2Eint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (12)$$

**Definition 13** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

**Definition 14** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0x \in A.27a.(\lambda V1y \in A.27b.V0x)$

**Definition 15** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.(\lambda V0f \in ((A.27c^{A.27b})^{A.27$

**Definition 16** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a}) A$

**Definition 17** We define  $c\_2Equotient\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f$

**Definition 18** We define  $c\_2Equotient\_2E\_3D\_3D\_3D\_3E$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27a})$

**Definition 19** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 20** We define  $c\_2Equotient\_2EQUOTIENT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda$

**Definition 21** We define  $c\_2Ecombin\_2EW$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x$

**Definition 22** We define  $c\_2Equotient\_2Erespects$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(c\_2Ecombin\_2EW A\_27a A\_27b$

**Definition 23** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x))$

**Definition 24** We define  $c\_2Ebool\_2ERES\_FORALL$  to be  $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).$

**Definition 25** We define  $c\_2Equotient\_2EEQUIV$  to be  $\lambda A\_27a : \iota.\lambda V0E \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2E$

Assume the following.

$$True \tag{13}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{16}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap (c\_2Ecombin\_2EI A\_27a) V0x) = V0x)) \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
& (\forall V1q \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
& ((p\ (ap\ (ap\ c\_2Einteger\_2Etint\_eq\ V0p)\ V1q)) \Leftrightarrow ((ap\ c\_2Einteger\_2Etint\_eq \\
& \quad V0p) = (ap\ c\_2Einteger\_2Etint\_eq\ V1q))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
& (\forall V1q \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \tag{20} \\
& ((V0p = V1q) \Rightarrow (p\ (ap\ (ap\ c\_2Einteger\_2Etint\_eq\ V0p)\ V1q))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
& (\forall V1y \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
& ((ap\ (ap\ c\_2Einteger\_2Etint\_add\ V0x)\ V1y) = (ap\ (ap\ c\_2Einteger\_2Etint\_add \\
& \quad V1y)\ V0x))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
& (\forall V1y \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
& (\forall V2z \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
& ((ap\ (ap\ c\_2Einteger\_2Etint\_add\ V0x)\ (ap\ (ap\ c\_2Einteger\_2Etint\_add \\
& \quad V1y)\ V2z)) = (ap\ (ap\ c\_2Einteger\_2Etint\_add\ (ap\ (ap\ c\_2Einteger\_2Etint\_add \\
& \quad V0x)\ V1y))\ V2z))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
& (\forall V1x2 \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
& (\forall V2y1 \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
& (\forall V3y2 \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
& (((p\ (ap\ (ap\ c\_2Einteger\_2Etint\_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c\_2Einteger\_2Etint\_eq \\
& \quad V2y1)\ V3y2))) \Rightarrow (p\ (ap\ (ap\ c\_2Einteger\_2Etint\_eq\ (ap\ (ap\ c\_2Einteger\_2Etint\_add \\
& \quad V0x1)\ V2y1))\ (ap\ (ap\ c\_2Einteger\_2Etint\_add\ V1x2)\ V3y2))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
& \quad ty\_2Enum\_2Enum)\ ty\_2Einteger\_2Eint)\ c\_2Einteger\_2Etint\_eq) \\
& \quad c\_2Einteger\_2Eint\_ABS)\ c\_2Einteger\_2Eint\_REP))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Einteger\_2Eint. ((ap\ (ap\ c\_2Einteger\_2Eint\_add \\
& \quad (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0))\ V0x) = V0x))
\end{aligned} \tag{25}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap (ap c\_2Einteger\_2Eint\_add (ap c\_2Einteger\_2Eint\_neg V0x)) V0x) = (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)))) \quad (26)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap (ap c\_2Einteger\_2Eint\_add V0x) (ap c\_2Einteger\_2Eint\_neg V0x)) = (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)))) \quad (27)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2z \in ty\_2Einteger\_2Eint.((p (ap (ap c\_2Einteger\_2Eint\_lt (ap (ap c\_2Einteger\_2Eint\_add V0x) V2z)) (ap (ap c\_2Einteger\_2Eint\_add V1y) V2z))) \Leftrightarrow (p (ap (ap c\_2Einteger\_2Eint\_lt V0x) V1y)))))) \quad (28)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT A\_27a A\_27a) (c\_2Emin\_2E\_3D A\_27a)) (c\_2Ecombin\_2EI A\_27a)) (c\_2Ecombin\_2EI A\_27a))) \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow \forall A\_27c. \\ & \quad nonempty A\_27c \Rightarrow \forall A\_27d.nonempty A\_27d \Rightarrow (\forall V0R1 \in ( \\ & \quad (2^{A\_27a} A\_27a).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\ & \quad (A\_27a^{A\_27c}).((p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT A\_27a A\_27c) \\ & \quad V0R1) V1abs1) V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b} A\_27b).(\forall V4abs2 \in \\ & \quad (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT \\ & \quad A\_27b A\_27d) V3R2) V4abs2) V5rep2)) \Rightarrow (p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT \\ & \quad (A\_27b^{A\_27a}) (A\_27d^{A\_27c})) (ap (ap (c\_2Equotient\_2E\_3D\_3D\_3E \\ & \quad A\_27a A\_27b) V0R1) V3R2)) (ap (ap (c\_2Equotient\_2E\_2D\_2D\_3E A\_27c \\ & \quad A\_27b A\_27a A\_27d) V2rep1) V4abs2)) (ap (ap (c\_2Equotient\_2E\_2D\_2D\_3E \\ & \quad A\_27a A\_27d A\_27c A\_27b) V1abs1) V5rep2)))))))))) \quad (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & \quad \forall V0R \in ((2^{A\_27a} A\_27a).(\forall V1abs \in (A\_27b^{A\_27a}). \\ & \quad (\forall V2rep \in (A\_27a^{A\_27b}).((p (ap (ap (ap (c\_2Equotient\_2EQUOTIENT \\ & \quad A\_27a A\_27b) V0R) V1abs) V2rep)) \Rightarrow (\forall V3x \in A\_27b.(\forall V4y \in \\ & \quad A\_27b.((V3x = V4y) \Leftrightarrow (p (ap (ap V0R (ap V2rep V3x)) (ap V2rep V4y)))))))))) \quad (31) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A\_27a.(\forall V4x2 \in \\
& \quad A\_27a.(\forall V5y1 \in A\_27a.(\forall V6y2 \in A\_27a.(((p\ (ap\ (ap\ V0R \\
& \quad V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2))))))))))))) \\
& \hspace{15em} (32)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\
& \quad (2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\
& (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in \\
& (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27d^{A\_27c}). \\
& ((\lambda V7x \in A\_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E \\
& \quad A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A\_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x))))))))))))) \\
& \hspace{15em} (33)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0REL \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A\_27a.(\forall V4x2 \in \\
& A\_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2))))))))))))) \\
& \hspace{15em} (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27b}).((p\ ( \\
& ap\ (c\_2Ebool\_2E\_21\ A\_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL \\
& \quad A\_27a)\ (ap\ (c\_2Equotient\_2Erespects\ A\_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& (c\_2Equotient\_2E\_2D\_2D\_3E\ A\_27a\ 2\ A\_27b\ 2)\ V1abs)\ (c\_2Ecombin\_2EI \\
& \quad 2))\ V3f)))))))))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A.27a})^{A.27a}).(\forall V1abs \in (A.27b^{A.27a}). \\
& (\forall V2rep \in (A.27a^{A.27b}).((p\ (ap\ (ap\ (ap\ (c.2Equotient.2EQUOTIENT \\
& \quad A.27a\ A.27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A.27a}).(\forall V4g \in \\
& \quad (2^{A.27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c.2Equotient.2E.3D.3D.3D.3E\ A.27a \\
& \quad 2)\ V0R)\ (c.2Emin.2E.3D\ 2)\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c.2Ebool.2ERES\_FORALL \\
& \quad A.27a)\ (ap\ (c.2Equotient.2Erespects\ A.27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ ( \\
& \quad ap\ (ap\ (c.2Ebool.2ERES\_FORALL\ A.27a)\ (ap\ (c.2Equotient.2Erespects \\
& \quad A.27a\ 2)\ V0R))\ V4g))))))))) \\
& \hspace{20em} (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0R1 \in ( \\
& \quad (2^{A.27a})^{A.27a}).(\forall V1abs1 \in (A.27c^{A.27a}).(\forall V2rep1 \in \\
& \quad (A.27a^{A.27c}).((p\ (ap\ (ap\ (ap\ (c.2Equotient.2EQUOTIENT\ A.27a\ A.27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A.27b})^{A.27b}).(\forall V4abs2 \in \\
& \quad (A.27d^{A.27b}).(\forall V5rep2 \in (A.27b^{A.27d}).((p\ (ap\ (ap\ (ap\ (c.2Equotient.2EQUOTIENT \\
& \quad A.27b\ A.27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A.27b^{A.27a}). \\
& \quad (\forall V7g \in (A.27b^{A.27a}).(\forall V8x \in A.27a.(\forall V9y \in \\
& \quad A.27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c.2Equotient.2E.3D.3D.3D.3E\ A.27a \\
& \quad A.27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ ( \\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))) \\
& \hspace{20em} (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0E \in ((2^{A.27a})^{A.27a}). \\
& \quad (\forall V1P \in (2^{A.27a}).((p\ (ap\ (c.2Equotient.2EEQUIV\ A.27a) \\
& \quad V0E)) \Rightarrow ((p\ (ap\ (ap\ (c.2Ebool.2ERES\_FORALL\ A.27a)\ (ap\ (c.2Equotient.2Erespects \\
& \quad A.27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c.2Ebool.2E.21\ A.27a)\ V1P)))))) \\
& \hspace{20em} (38)
\end{aligned}$$

### Theorem 1

$$\begin{aligned}
& (\forall V0x \in ty.2Einteger.2Eint.(\forall V1y \in ty.2Einteger.2Eint. \\
& \quad ((p\ (ap\ (ap\ c.2Einteger.2Eint\_lt\ (ap\ c.2Einteger.2Eint\_neg \\
& \quad V0x))\ (ap\ c.2Einteger.2Eint\_neg\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ c.2Einteger.2Eint\_lt \\
& \quad V1y)\ V0x))))))
\end{aligned}$$