

# thm\_2Einteger\_2EINT\_\_LT\_\_SUB\_\_RADD (TMQQ384LneMW6RDE7GqyLYpKozE8LiMXoby)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{3}$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint}) \tag{4}$$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^{A.^{27a}}).(ap (ap (c\_2Emin\_2E\_3D (2^{A.^{27a}}))$

**Definition 5** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E\_40 (ty\_2Einteger\_2Eint$

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \tag{5}$$

**Definition 6** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint$

Let  $c\_2Einteger\_2Etint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum) (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)) \quad (6)$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)) \quad (7)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})} \quad (8)$$

**Definition 7** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$

**Definition 8** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap\ c\_2Einteger\_2Eint\_neg\ T1)$

Let  $c\_2Einteger\_2Etint\_add : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum) (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)) (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)) \quad (9)$$

**Definition 9** We define  $c\_2Einteger\_2Eint\_add$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint.$

**Definition 10** We define  $c\_2Einteger\_2Eint\_sub$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2Eint.$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint. \\ & (\forall V2z \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt \\ & (ap\ (ap\ c\_2Einteger\_2Eint\_add\ V0x)\ V2z))\ (ap\ (ap\ c\_2Einteger\_2Eint\_add \\ & V1y)\ V2z))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ V0x)\ V1y)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint. \\ & ((ap\ (ap\ c\_2Einteger\_2Eint\_add\ (ap\ (ap\ c\_2Einteger\_2Eint\_sub \\ & V0x)\ V1y))\ V1y) = V0x))) \end{aligned} \quad (13)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint. \\ & (\forall V2z \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt \\ & (ap\ (ap\ c\_2Einteger\_2Eint\_sub\ V0x)\ V1y))\ V2z)) \Leftrightarrow (p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt \\ & V0x)\ (ap\ (ap\ c\_2Einteger\_2Eint\_add\ V2z)\ V1y)))))) \end{aligned}$$