thm_2Einteger_2EINT__MOD__CALCULATE (TM-REU81BvT4Jv5G5DneVQ3R3xyq5MQASjho)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{3}$$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{4}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{5}$$

Definition 6 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 7 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 $(ty$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{6}$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{7}$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \tag{8}$$

Definition 8 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 9 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint__

Let $c\_2Einteger\_2Eint\_\_mod : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_mod \in ((ty\_2Einteger\_2Eint^{ty\_2Einteger\_2Eint})^{ty\_2Einteger\_2Eint}) \tag{9}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{10}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{11}$$

Definition 10 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \tag{12}$$

Definition 11 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 12 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x)) = V0x)) \tag{13}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(V0x = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))\tag{14}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n))\Leftrightarrow(V0m = V1n))))\tag{15}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.((\neg(V1m = \mathsf{c\_2Enum\_2E0}))\Rightarrow((ap\ (ap\ c\_2Einteger\_2Eint\_\_mod\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0n)\ V1m))))))\tag{16}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1q \in ty\_2Einteger\_2Eint.((\neg(V1q = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Rightarrow((ap\ (ap\ c\_2Einteger\_2Eint\_\_mod\ V0p)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1q)) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ c\_2Einteger\_2Eint\_\_mod\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0p))\ V1q))))))\tag{17}$$

Theorem 1

$$((\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.((\neg(V1m = \mathsf{c\_2Enum\_2E0}))\Rightarrow((ap\ (ap\ c\_2Einteger\_2Eint\_\_mod\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0n)\ V1m))))))\land((\forall V2p \in ty\_2Einteger\_2Eint.(\forall V3q \in ty\_2Einteger\_2Eint.((\neg(V3q = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Rightarrow((ap\ (ap\ c\_2Einteger\_2Eint\_\_mod\ V2p)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V3q)) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ c\_2Einteger\_2Eint\_\_mod\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V2p))\ V3q))))))\land((\forall V4x \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V4x)) = V4x))\land((\forall V5m \in ty\_2Enum\_2Enum.(\forall V6n \in ty\_2Enum\_2Enum.(((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V5m) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V6n))\Leftrightarrow(V5m = V6n))))\land(\forall V7x \in ty\_2Einteger\_2Eint.(((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V7x) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(V7x = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))))))$$