

thm\_2Einteger\_2EINT\_\_MOD\_\_ID  
(TMJgZBAZZk8KRSJFWVdA39KpSi8Gn79A6ky)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_Ebool\_2ET$  to be  $(ap \ (ap \ (c\_Emin\_2E\_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

*nonempty* *ty\_2Enum\_2Enum* (2)

Let  $c_2Enum_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (3)$$

**Definition 3** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 4** We define  $\text{c\_2Earithmetic\_2EZERO}$  to be  $\text{c\_Enum\_2E0}$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^\omega) \quad (5)$$

**Definition 5** We define  $c_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^A\_{27a}).(ap\ (ap\ (c_2Emin\_2E\_3D\ (2^A\_{27a}\ P)\ V)\ 0)\ P))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ ($

Let  $c_2Earithmetic_2E_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT1 n) 0)$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty\ A0 \Rightarrow & \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod \\ & A0\ A1) \end{aligned} \tag{7}$$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{8}$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})ty\_2Einteger\_2Eint) \tag{9}$$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p \text{ of type } \iota \Rightarrow \iota))$

**Definition 10** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E\_40 (ty\_2Einteger\_2Eint a)))$

Let  $c\_2Einteger\_2Etint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)) \tag{10}$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum) \tag{11}$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})} \tag{12}$$

**Definition 11** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).r$

**Definition 12** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap c\_2Einteger\_2Eint\_neg T1)$

Let  $c\_2Einteger\_2Eint\_mod : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_mod \in ((ty\_2Einteger\_2Eint)^{ty\_2Einteger\_2Eint})ty\_2Einteger\_2Eint \tag{13}$$

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum) \tag{14}$$

**Definition 13** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint. \lambda V1T2 \in ty\_2Einteger.$

**Definition 14** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 15** We define  $c_{\text{2Emin\_3D\_3D\_3E}}$  to be  $\lambda P \in 2.\lambda Q \in 2.\text{inj\_o} (p \Rightarrow p Q)$  of type  $\iota$ .

**Definition 16** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E))$

**Definition 17** We define  $c\_2Einteger\_2Eint\_le$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint. \lambda V1y \in ty\_2Einteger\_2Eint.$

Let  $c : \text{Einteger}$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (15)$$

**Definition 19** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A.\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.\_27a.(\lambda V2t2 \in A.\_27a.($

Let  $c_2Einteger_2Etint\_mul : \iota$  be given. Assume the following.

$c\_2Einteger\_2Etint\_mul \in (((ty\_2Epol_1 \rightarrow (t_1 \cdot 2Epol_1 \rightarrow 2Epol_1) \rightarrow t_1 \cdot 2Epol_1 \rightarrow 2Epol_1) \rightarrow t_1 \cdot 2Epol_1 \rightarrow 2Epol_1) \rightarrow t_1 \cdot 2Epol_1 \rightarrow 2Epol_1)$

**Definition 20** We define  $c\_2Einteger\_2Eint\_mul$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint. \lambda V1T2 \in ty\_2Einteger_2Eint.$

Let  $c_2$ ,  $\text{Einteger}_2$ ,  $E\text{tint\_add}$  :  $\iota$  be given. Assume the following.

*c* 2Einteger 2Etint add ∈ (((tu 2Epc

**Definition 21** We define  $\mathcal{C}_2\text{Finteger}.\mathcal{C}_2\text{Fint}.\text{add\_to}$  to be  $\lambda V0T1 \in \mathit{ty\_2Finteger}.\mathit{2Fint}.\lambda V1T2 \in \mathit{ty\_2Finteger}.$

D. S. Vile - CS, Week 6 - 25E + 25E + 1, N, 4, 27 - (NH<sub>2</sub>)<sub>2</sub>CO<sub>2</sub> (24, 37a) & - (NH<sub>2</sub>)<sub>2</sub>CO<sub>2</sub> (6, 25) in 25E + 12

Answer the following.

$$((\forall x1 \in L, (\forall y2 \in L, (((p \vee x1) \wedge (p \vee y2)) \rightarrow ((p \vee 1t2) \Rightarrow (p \vee 0t1)) \Rightarrow ((p \vee 0t1) \Leftrightarrow (p \vee 1t2))))))) \quad (19)$$

Assume the following:

$$(\forall \forall \alpha \in \omega. (T \text{ false} \rightarrow (p \vee \alpha))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & \quad ((p V0t) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & \quad (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & \quad (p V0t)) \Leftrightarrow (p V0t))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & \quad True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & \quad (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \end{aligned} \quad (26)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))) \quad (27)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ True)) \quad (28)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & \quad ((p V0t) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & \quad (p V0t))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A_27a. & nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\ V0t1) V1t2) = V1t2)))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee ( \\ p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg( \\ p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (34)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A_27a. & nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ V5y_27))))))) \end{aligned} \quad (36)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add \\ V0x) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) = V0x)) \quad (37)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\ (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\ (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x)) \quad (38)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint. (p (ap (ap c_2Einteger_2Eint_le \\ V0x) V0x))) \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Einteger\_2Eint. ((p (ap (ap c\_2Einteger\_2Eint\_lt \\ (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) (ap c\_2Einteger\_2Eint\_neg \\ V0x))) \Leftrightarrow (p (ap (ap c\_2Einteger\_2Eint\_lt V0x) (ap c\_2Einteger\_2Eint\_of\_num \\ c\_2Enum\_2E0)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Einteger\_2Eint. ((V0x = (ap c\_2Einteger\_2Eint\_of\_num \\ c\_2Enum\_2E0)) \vee ((p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_of\_num \\ c\_2Enum\_2E0)) V0x)) \vee (p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_of\_num \\ c\_2Enum\_2E0)) (ap c\_2Einteger\_2Eint\_neg V0x))))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0i \in ty\_2Einteger\_2Eint. (\forall V1j \in ty\_2Einteger\_2Eint. \\ & (\forall V2m \in ty\_2Einteger\_2Eint. ((\exists V3q \in ty\_2Einteger\_2Eint. \\ & ((V0i = (ap (ap c\_2Einteger\_2Eint\_add (ap (ap c\_2Einteger\_2Eint\_mul \\ V3q) V1j)) V2m)) \wedge (p (ap (ap (c\_2Ebool\_2ECOND 2) (ap (ap c\_2Einteger\_2Eint\_lt \\ V1j) (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0))) (ap (ap c\_2Ebool\_2E\_2F\_5C \\ (ap (ap c\_2Einteger\_2Eint\_lt V1j) V2m)) (ap (ap c\_2Einteger\_2Eint\_le \\ V2m) (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0))) (ap (ap c\_2Ebool\_2E\_2F\_5C \\ (ap (ap c\_2Einteger\_2Eint\_le (ap c\_2Einteger\_2Eint\_of\_num \\ c\_2Enum\_2E0)) V2m)) (ap (ap c\_2Einteger\_2Eint\_lt V2m) V1j)))))) \Rightarrow \\ & ((ap (ap c\_2Einteger\_2Eint\_mod V0i) V1j) = V2m))))))) \end{aligned} \quad (42)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \end{aligned} \quad (46)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (47)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
 & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
 & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
 & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
 & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
 & (\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{51}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
 (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \tag{52}$$

### Theorem 1

$$\begin{aligned}
 & (\forall V0i \in ty\_2Einteger\_2Eint. ((\neg(V0i = (ap c\_2Einteger\_2Eint\_of\_num \\
 & c\_2Enum\_2E0))) \Rightarrow ((ap (ap c\_2Einteger\_2Eint\_mod V0i) V0i) = (ap \\
 & c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)))) \\
 \end{aligned}$$