

thm_2Einteger_2EINT_MOD_PLUS

(TMWS5GENyjoBQm5pBgmWY7puiBsAWbET2iD)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod$$

$$A0\ A1)$$

(2)

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)} \quad (3)$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (4)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint})^{(ty_2Einteger_2Eint)} \quad (5)$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a}))))$

Definition 5 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint. (ap (c_2Emin_2E_40 (ty_2Einteger_2Eint)))$

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}}) \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})})^{\text{ty_2Epair_2Eprod ty_2Enum_2Enum}} \quad (7)$$

Definition 6 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum)$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{\text{ty_2Epair_2Eprod ty_2Enum}})^{\text{ty_2Epair_2Eprod ty_2Enum}} \quad (8)$$

Definition 7 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. \lambda V1T2 \in ty_2Einteger_2Eint$

Definition 8 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E))$

Definition 11 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint. \lambda V1y \in ty_2Einteger_2Eint$

Let $c_2Einteger_2Eint_div : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_div \in ((ty_2Einteger_2Eint^{ty_2Einteger_2Eint})^{\text{ty_2Einteger_2Eint}})^{\text{ty_2Einteger_2Eint}} \quad (9)$$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}}) \quad (10)$$

Definition 12 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. \lambda V1T2 \in ty_2Einteger_2Eint$

Definition 13 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. \lambda V1T2 \in ty_2Einteger_2Eint$

Let $c_2Einteger_2Eint_mod : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_mod \in ((ty_2Einteger_2Eint^{ty_2Einteger_2Eint})^{\text{ty_2Einteger_2Eint}})^{\text{ty_2Einteger_2Eint}} \quad (11)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (13)$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2 \in \text{integer_of_num} \in (\text{ty_integer_int}^{\text{ty_enum_enum}}) \quad (14)$$

Definition 15 We define $\text{c_2Emarker_2EAbbrev}$ to be $\lambda V0x \in 2.V0x$.

Definition 16 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 17 We define $c_2E\text{Marker}_2E\text{AC}$ to be $\lambda V0b1 \in 2.\lambda V1b2 \in 2.(ap\ (ap\ c_2E\text{bool}_2E\text{F}_5C\ V0b1)$

Definition 18 We define $c_Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 19 We define c_2 Ecombin_2ES to be $\lambda A. \lambda 27a : \iota. \lambda A. \lambda 27b : \iota. \lambda A. \lambda 27c : \iota. (\lambda V0 f \in ((A. \lambda 27c)^{A \rightarrow 27b})^A. 2$

Definition 20 We define $c_2Ecombin_2EI$ to be $\lambda A._27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A._27a\ (A._27a^A._27a))\ A)\ A)$

Definition 21 We define c_2 to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda A.27d : \iota.\lambda V.0$.

Definition 22 We define c to be the quotient of $2E$, $3D$, $3D$, $3D$, $3E$ to be $\lambda A. 27a : \nu. \lambda A. 27b : \nu. \lambda V0R1 \in ((2^{A-27a})^A)^{A-27b}$

Definition 23. We define c -2Equotient, 2EQUOTIENT to be $\lambda A. \lambda 27a : \lambda A. \lambda 27b : \lambda V0B \in ((2^{A \cdot 27a})^{\cdot 2^{A \cdot 27a}})$.

Definition 24. We define a 2E-combin-2EW type to be $\lambda A. 27a \rightarrow \lambda A. 27b \rightarrow ((\lambda V. Vf) ((\lambda A. 27b) A^{27a}) A^{27a})$. $(\lambda V. Vg) ((\lambda A. 27b) A^{27a}) A^{27a}$.

Definition 25. We define a 2Equitant-2F respects to be $\lambda_4 \cdot 37a + \lambda_4 \cdot 37b + (\alpha \cdot 2Equi-2EW \cdot 4 \cdot 37c - 4 \cdot 37d)$

Definition 26. We define a 2EhooL 2EFIN to be a 4-27a : $\lambda V0m \in 4\text{-}27a$ ($\forall 1f \in (2A\text{-}27a)$) ($\exists m V1f$) $V0m$

Definition 37. We define a 2Ehool 2EFBES EFORALL_t to be $\lambda A. \exists^{\leq t} a : A. (\forall b : A. \exists^{\leq t} c : A. b =_{\text{EFBES}} c) \wedge (\forall m : M. \exists^{\leq t} n : M. m =_{\text{EFBES}} n)$.

D, G, H = 22, W, L, S = 25E + 1.5E-5G, 2E + 1.1 (NUC1 = 2 (NUC2 = 2.6 + (-2E1 + 1.5E-21)G) (NUC3 =

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$$((\forall v t1 \in Z. (\forall v t2 \in Z. ((p \vee v t1) \rightarrow (p \vee v t2)) \rightarrow (((p \vee 1 t2) \Rightarrow (p \vee 0 t1)) \Rightarrow ((p \vee 0 t1) \Leftrightarrow (p \vee 1 t2))))))) \quad (16)$$

Assume the following:

$$(\forall V \otimes t \in \Sigma. (T \text{ false} \rightarrow (p \vee \otimes t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (18)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (19)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (20)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (23)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V1P V3x)) \vee (p V0Q))))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A))))))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (27)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0f \in ((A_27a^{A_27a})^{A_27a}). \\
& \quad ((\forall V1x \in A_27a.(\forall V2y \in A_27a.(\forall V3z \in A_27a. \\
& \quad ((ap (ap V0f V1x) (ap (ap V0f V2y) V3z)) = (ap (ap V0f (ap (ap V0f V1x) \\
& \quad V2y)) V3z)))))) \Rightarrow ((\forall V4x \in A_27a.(\forall V5y \in A_27a.((ap \\
& \quad (ap V0f V4x) V5y) = (ap (ap V0f V5y) V4x)))) \Rightarrow (\forall V6x \in A_27a. \\
& \quad (\forall V7y \in A_27a.(\forall V8z \in A_27a.((ap (ap V0f V6x) (ap (ap \\
& \quad V0f V7y) V8z)) = (ap (ap V0f V7y) (ap (ap V0f V6x) V8z))))))) \\
& \quad (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Ecombin_2El \\
& \quad A_27a) V0x) = V0x)) \\
& \quad (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& \quad (\forall V1q \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& \quad ((p (ap (ap c_2Einteger_2Etint_eq V0p) V1q)) \Leftrightarrow ((ap c_2Einteger_2Etint_eq \\
& \quad V0p) = (ap c_2Einteger_2Etint_eq V1q)))) \\
& \quad (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& \quad (\forall V1q \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& \quad ((V0p = V1q) \Rightarrow (p (ap (ap c_2Einteger_2Etint_eq V0p) V1q)))) \\
& \quad (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& \quad (\forall V1y \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& \quad (\forall V2z \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& \quad ((ap (ap c_2Einteger_2Etint_add V0x) (ap (ap c_2Einteger_2Etint_add \\
& \quad V1y) V2z)) = (ap (ap c_2Einteger_2Etint_add (ap (ap c_2Einteger_2Etint_add \\
& \quad V0x) V1y)) V2z)))) \\
& \quad (32)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& \quad (\forall V1x2 \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& \quad (\forall V2y1 \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& \quad (\forall V3y2 \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& \quad (((p (ap (ap c_2Einteger_2Etint_eq V0x1) V1x2)) \wedge (p (ap (ap c_2Einteger_2Etint_eq \\
& \quad V2y1) V3y2))) \Rightarrow (p (ap (ap c_2Einteger_2Etint_eq (ap (ap c_2Einteger_2Etint_add \\
& \quad V0x1) V2y1)) (ap (ap c_2Einteger_2Etint_add V1x2) V3y2))))))) \\
& \quad (33)
\end{aligned}$$

Assume the following.

$$(p (ap (ap (ap (c_2Equotient_2EQUOTIENT (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum) ty_2Einteger_2Eint) c_2Einteger_2Etint_eq) c_2Einteger_2Eint_ABS) c_2Einteger_2Eint REP))) \quad (34)$$

Assume the following.

$$(\forall V0y \in ty_2Einteger_2Eint. (\forall V1x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add V1x) V0y) = (ap (ap c_2Einteger_2Eint_add V0y) V1x)))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0q \in ty_2Einteger_2Eint. ((\neg(V0q = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \Rightarrow (\forall V1p \in ty_2Einteger_2Eint. ((V1p = (ap (ap c_2Einteger_2Eint_add (ap (ap c_2Einteger_2Eint_mul (ap (ap c_2Einteger_2Eint_div V1p) V0q)) (ap (ap c_2Einteger_2Eint_mod V1p) V0q))) \wedge (p (ap (ap (c_2Ebool_2ECOND 2) (ap (ap c_2Einteger_2Eint_lt V0q) (ap (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) (ap (ap c_2Ebool_2E_2F_5C (ap (ap c_2Einteger_2Eint_le (ap (ap c_2Einteger_2Eint_mod V1p) V0q)) (ap (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)))) (ap (ap c_2Ebool_2E_2F_5C (ap (ap c_2Einteger_2Eint_le (ap (ap c_2Einteger_2Eint_mod V1p) V0q))) (ap (ap c_2Einteger_2Eint_lt (ap (ap c_2Einteger_2Eint_mod V1p) V0q))))))))))) \quad (36) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0k \in ty_2Einteger_2Eint. (\forall V1q \in ty_2Einteger_2Eint. \\ & (\forall V2r \in ty_2Einteger_2Eint. ((\neg(V0k = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \Rightarrow ((ap (ap c_2Einteger_2Eint_mod (ap (ap c_2Einteger_2Eint_add (ap (ap c_2Einteger_2Eint_mul V1q) V0k)) V2r)) V0k) = (ap (ap c_2Einteger_2Eint_mod V2r) V0k))))))) \quad (37) \end{aligned}$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (p (ap (ap (ap (c_2Equotient_2EQUOTIENT A_27a A_27a) (c_2Emin_2E_3D A_27a)) (c_2Ecombin_2EI A_27a)) (c_2Ecombin_2EI A_27a))) \quad (38)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \forall A_{27c}. \\
& nonempty A_{27c} \Rightarrow \forall A_{27d}.nonempty A_{27d} \Rightarrow (\forall V0R1 \in (\\
& (2^{A_{27a}})^{A_{27a}}).(\forall V1abs1 \in (A_{27c})^{A_{27a}}).(\forall V2rep1 \in \\
& (A_{27a})^{A_{27c}}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT A_{27a} A_{27c}) \\
& V0R1) V1abs1) V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_{27b}})^{A_{27b}}).(\forall V4abs2 \in \\
& (A_{27d})^{A_{27b}}).(\forall V5rep2 \in (A_{27b})^{A_{27d}}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& A_{27b} A_{27d}) V3R2) V4abs2) V5rep2)) \Rightarrow (p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& (A_{27b})^{A_{27a}}) (A_{27d})^{A_{27c}}) (ap (ap (c_2Equotient_2E_3D_3D_3D_3E \\
& A_{27a} A_{27b}) V0R1) V3R2)) (ap (ap (c_2Equotient_2E_2D_2D_3E A_{27c} \\
& A_{27b} A_{27a} A_{27d}) V2rep1) V4abs2)) (ap (ap (c_2Equotient_2E_2D_2D_3E \\
& A_{27a} A_{27d} A_{27c} A_{27b}) V1abs1) V5rep2))))))))))) \\
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow (\\
& \forall V0R \in ((2^{A_{27a}})^{A_{27a}}).(\forall V1abs \in (A_{27b})^{A_{27a}}). \\
& (\forall V2rep \in (A_{27a})^{A_{27b}}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& A_{27a} A_{27b}) V0R) V1abs) V2rep)) \Rightarrow (\forall V3x \in A_{27b}.(\forall V4y \in \\
& A_{27b}.((V3x = V4y) \Leftrightarrow (p (ap (ap V0R (ap V2rep V3x)) (ap V2rep V4y)))))))) \\
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow (\\
& \forall V0R \in ((2^{A_{27a}})^{A_{27a}}).(\forall V1abs \in (A_{27b})^{A_{27a}}). \\
& (\forall V2rep \in (A_{27a})^{A_{27b}}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& A_{27a} A_{27b}) V0R) V1abs) V2rep)) \Rightarrow (\forall V3x1 \in A_{27a}.(\forall V4x2 \in \\
& A_{27a}.(\forall V5y1 \in A_{27a}.(\forall V6y2 \in A_{27a}.((p (ap (ap V0R \\
& V3x1) V4x2)) \wedge (p (ap (ap V0R V5y1) V6y2))) \Rightarrow ((p (ap (ap V0R V3x1) V5y1) \\
& (p (ap (ap V0R V4x2) V6y2))))))))))) \\
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \forall A_{27c}. \\
& nonempty A_{27c} \Rightarrow \forall A_{27d}.nonempty A_{27d} \Rightarrow (\forall V0R1 \in (\\
& (2^{A_{27a}})^{A_{27a}}).(\forall V1abs1 \in (A_{27c})^{A_{27a}}).(\forall V2rep1 \in \\
& (A_{27a})^{A_{27c}}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT A_{27a} A_{27c}) \\
& V0R1) V1abs1) V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_{27b}})^{A_{27b}}).(\forall V4abs2 \in \\
& (A_{27d})^{A_{27b}}).(\forall V5rep2 \in (A_{27b})^{A_{27d}}).((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& A_{27b} A_{27d}) V3R2) V4abs2) V5rep2)) \Rightarrow (\forall V6f \in (A_{27d})^{A_{27c}}). \\
& ((\lambda V7x \in A_{27c}.(ap V6f V7x)) = (ap (ap (ap (c_2Equotient_2E_2D_2D_3E \\
& A_{27c} A_{27b} A_{27a} A_{27d}) V2rep1) V4abs2) (\lambda V8x \in A_{27a}.(ap V5rep2 \\
& (ap V6f (ap V1abs1 V8x))))))))))) \\
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \\
& \quad \forall V0REL \in ((2^{A_{.27a}})^{A_{.27a}}).(\forall V1abs \in (A_{.27b})^{A_{.27a}}). \\
& \quad (\forall V2rep \in (A_{.27a})^{A_{.27b}}).((p (ap (ap (ap (c_{.2Equotient_2EQUOTIENT} \\
& \quad A_{.27a} A_{.27b}) V0REL) V1abs) V2rep)) \Rightarrow (\forall V3x1 \in A_{.27a}.(\forall V4x2 \in \\
& \quad A_{.27a}.((p (ap (ap V0REL V3x1) V4x2)) \Rightarrow (p (ap (ap V0REL V3x1) (ap V2rep \\
& \quad (ap V1abs V4x2))))))))))) \\
& \tag{43}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \\
& \quad \forall V0R \in ((2^{A_{.27a}})^{A_{.27a}}).(\forall V1abs \in (A_{.27b})^{A_{.27a}}). \\
& \quad (\forall V2rep \in (A_{.27a})^{A_{.27b}}).((p (ap (ap (ap (c_{.2Equotient_2EQUOTIENT} \\
& \quad A_{.27a} A_{.27b}) V0R) V1abs) V2rep)) \Rightarrow (\forall V3f \in (2^{A_{.27b}}).((p (\\
& \quad ap (c_{.2Ebool_2E_21} A_{.27b}) V3f)) \Leftrightarrow (p (ap (ap (c_{.2Ebool_2ERES_FORALL} \\
& \quad A_{.27a}) (ap (c_{.2Equotient_2Erespects} A_{.27a} 2) V0R)) (ap (ap (ap \\
& \quad (c_{.2Equotient_2E_2D_2D_3E} A_{.27a} 2 A_{.27b} 2) V1abs) (c_{.2Ecombin_2E} \\
& \quad 2)) V3f))))))))))) \\
& \tag{44}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \\
& \quad \forall V0R \in ((2^{A_{.27a}})^{A_{.27a}}).(\forall V1abs \in (A_{.27b})^{A_{.27a}}). \\
& \quad (\forall V2rep \in (A_{.27a})^{A_{.27b}}).((p (ap (ap (c_{.2Equotient_2EQUOTIENT} \\
& \quad A_{.27a} A_{.27b}) V0R) V1abs) V2rep)) \Rightarrow (\forall V3f \in (2^{A_{.27a}}).(\forall V4g \in \\
& \quad (2^{A_{.27a}}).((p (ap (ap (ap (c_{.2Equotient_2E_3D_3D_3E} A_{.27a} \\
& 2) V0R) (c_{.2Emin_2E_3D} 2)) V3f) V4g)) \Rightarrow ((p (ap (ap (c_{.2Ebool_2ERES_FORALL} \\
& \quad A_{.27a}) (ap (c_{.2Equotient_2Erespects} A_{.27a} 2) V0R)) V3f)) \Leftrightarrow (p (\\
& \quad ap (ap (c_{.2Ebool_2ERES_FORALL} A_{.27a}) (ap (c_{.2Equotient_2Erespects} \\
& \quad A_{.27a} 2) V0R)) V4g))))))))))) \\
& \tag{45}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}. \\
& \quad nonempty A_{.27c} \Rightarrow \forall A_{.27d}.nonempty A_{.27d} \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_{.27a}})^{A_{.27a}}).(\forall V1abs1 \in (A_{.27c})^{A_{.27a}}).(\forall V2rep1 \in \\
& \quad (A_{.27a})^{A_{.27c}}).((p (ap (ap (c_{.2Equotient_2EQUOTIENT} A_{.27a} A_{.27c}) \\
& \quad V0R1) V1abs1) V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_{.27b}})^{A_{.27b}}).(\forall V4abs2 \in \\
& \quad (A_{.27d})^{A_{.27b}}).(\forall V5rep2 \in (A_{.27b})^{A_{.27d}}).((p (ap (ap (c_{.2Equotient_2EQUOTIENT} \\
& \quad A_{.27b} A_{.27d}) V3R2) V4abs2) V5rep2)) \Rightarrow (\forall V6f \in (A_{.27b})^{A_{.27a}}). \\
& \quad (\forall V7g \in (A_{.27b})^{A_{.27a}}).(\forall V8x \in A_{.27a}.(\forall V9y \in \\
& \quad A_{.27a}.(((p (ap (ap (ap (c_{.2Equotient_2E_3D_3D_3E} A_{.27a} \\
& \quad A_{.27b}) V0R1) V3R2) V6f) V7g)) \wedge (p (ap (ap V0R1 V8x) V9y))) \Rightarrow (p (ap (\\
& \quad ap V3R2 (ap V6f V8x)) (ap V7g V9y))))))))))) \\
& \tag{46}
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a. \text{nonempty } A_27a \Rightarrow & (\forall V0E \in ((2^{A_27a})^{A_27a}). \\ & (\forall V1P \in (2^{A_27a}).((p (ap (c_2\text{Equotient_2EEQUIV } A_27a) \\ & V0E)) \Rightarrow ((p (ap (ap (c_2\text{Ebool_2ERES_FORALL } A_27a) (ap (c_2\text{Equotient_2Erespects } \\ & A_27a 2) V0E)) V1P)) \Leftrightarrow (p (ap (c_2\text{Ebool_2E_21 } A_27a) V1P))))))) \end{aligned} \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (49)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \end{aligned} \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (52)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (55)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (57)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (\forall V3s \in 2. \\ & 2. (((p V0p) \Leftrightarrow (p (ap (ap (ap (c_2Ebool_2ECOND 2) V1q) V2r) V3s))) \Leftrightarrow \\ & (((p V0p) \vee ((p V1q) \vee (\neg(p V3s)))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge \\ & (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V3s)))) \wedge (((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))) \wedge \\ & ((p V1q) \vee ((p V3s) \vee (\neg(p V0p)))))))))))))) \end{aligned} \quad (58)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (63)$$

Theorem 1

$$\begin{aligned} & (\forall V0k \in ty_2Einteger_2Eint. (\forall V1i \in ty_2Einteger_2Eint. \\ & (\forall V2j \in ty_2Einteger_2Eint. ((\neg(V0k = (ap c_2Einteger_2Eint_of_num \\ & c_2Enum_2E0))) \Rightarrow ((ap (ap c_2Einteger_2Eint_mod (ap (ap c_2Einteger_2Eint_add \\ & (ap (ap c_2Einteger_2Eint_mod V1i) V0k)) (ap (ap c_2Einteger_2Eint_mod \\ & V2j) V0k)) V0k) = (ap (ap c_2Einteger_2Eint_mod (ap (ap c_2Einteger_2Eint_add \\ & V1i) V2j)) V0k))))))) \end{aligned}$$