

thm_2Einteger_2EINT__MOD__UNIQUE
(TMMh23L8VaLaPfBhiPZwV5hMe4bATqBu2KV)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Eenum_2E_enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2E_enum \tag{1}$$

Let $ty_2Epair_2E_eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2E_eprod\ A0\ A1) \tag{2}$$

Let $c_2Einteger_2E_etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2E_etint_eq \in ((2^{(ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum)})^{(ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum)}) \tag{3}$$

Let $ty_2Einteger_2E_eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2E_eint \tag{4}$$

Let $c_2Einteger_2E_eint_abs_class : \iota$ be given. Assume the following.

$$c_2Einteger_2E_eint_abs_class \in (ty_2Einteger_2E_eint)^{(2^{(ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum)})} \tag{5}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))))$

Definition 4 We define $c_2Einteger_2E_eint_abs$ to be $\lambda V0r \in (ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum)$

Let $c_2Einteger_2E_eint_rep_class : \iota$ be given. Assume the following.

$$c_2Einteger_2E_eint_rep_class \in ((2^{(ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum)})^{(ty_2Einteger_2E_eint)}} \tag{6}$$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 6 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty_2Einteger_2Eint_REP$

Let $c_2Einteger_2Eint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})(ty_2Epair_2Eprod ty_2Enum_2Enum)) \quad (7)$$

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (8)$$

Definition 7 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint_neg$

Let $c_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (9)$$

Definition 8 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 9 We define $c_2Einteger_2Eint_sub$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$

Let $c_2Einteger_2Eint_mod : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_mod \in ((ty_2Einteger_2Eint^{ty_2Einteger_2Eint})^{ty_2Einteger_2Eint}) \quad (10)$$

Let $c_2Einteger_2Eint_div : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_div \in ((ty_2Einteger_2Eint^{ty_2Einteger_2Eint})^{ty_2Einteger_2Eint}) \quad (11)$$

Definition 10 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$
of type ι .

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

Definition 14 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (25)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in A_27a. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p (ap V0P V2x)))))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_27a. (p (ap V1P V3x))) \vee (p V0Q)))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. (\forall V3y \in A_27a. ((ap V0f (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1b) V2x) V3y)) = (ap (ap (ap (c_2Ebool_2ECOND A_27b) V1b) (ap V0f V2x)) (ap V0f V3y)))))) \quad (37)$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in 2. (\forall V1t1 \in 2. (\forall V2t2 \in 2. ((p (ap (ap \\
& (ap (c.2Ebool_2ECOND 2) V0b) V1t1) V2t2)) \Leftrightarrow (((\neg(p V0b)) \vee (p V1t1)) \wedge \\
& ((p V0b) \vee (p V2t2))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c.2Ebool_2ECOND A_27a) \\
& V0P) V2x) V4y) = (ap (ap (ap (c.2Ebool_2ECOND A_27a) V1Q) V3x_27) \\
& V5y_27))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((ap (c.2Ecombin_2EI \\
A_27a) V0x) = V0x)) \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& (\forall V1q \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& ((p (ap (ap c.2Einteger_2Etint_eq V0p) V1q)) \Leftrightarrow ((ap c.2Einteger_2Etint_eq \\
& V0p) = (ap c.2Einteger_2Etint_eq V1q))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& (\neg(p (ap (ap c.2Einteger_2Etint_lt V0x) V0x))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& (\forall V1x2 \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& (\forall V2y1 \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& (\forall V3y2 \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& (((p (ap (ap c.2Einteger_2Etint_eq V0x1) V1x2)) \wedge (p (ap (ap c.2Einteger_2Etint_eq \\
& V2y1) V3y2)))) \Rightarrow ((p (ap (ap c.2Einteger_2Etint_lt V0x1) V2y1)) \Leftrightarrow \\
& (p (ap (ap c.2Einteger_2Etint_lt V1x2) V3y2))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (p (ap (ap (ap (ap (c.2Equotient_2EQUOTIENT (ty_2Epair_2Eprod ty_2Enum_2Enum \\
& ty_2Enum_2Enum) ty_2Einteger_2Eint) c.2Einteger_2Etint_eq) \\
& c.2Einteger_2Eint_ABS) c.2Einteger_2Eint_REP))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. (((p (ap (ap c_2Einteger_2Eint_le \\
V0x) V1y)) \wedge (p (ap (ap c_2Einteger_2Eint_lt V1y) V2z))) \Rightarrow (p (ap \\
& (ap c_2Einteger_2Eint_lt V0x) V2z))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap (ap c_2Einteger_2Eint_sub (ap (ap c_2Einteger_2Eint_add \\
V0x) V1y)) V0x) = V1y)))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0i \in ty_2Einteger_2Eint. (\forall V1j \in ty_2Einteger_2Eint. \\
& ((\neg (V1j = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \Rightarrow ((ap \\
& (ap c_2Einteger_2Eint_mod V0i) V1j) = (ap (ap c_2Einteger_2Eint_sub \\
V0i) (ap (ap c_2Einteger_2Eint_mul (ap (ap c_2Einteger_2Eint_div \\
& V0i) V1j)) V1j))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0i \in ty_2Einteger_2Eint. (\forall V1j \in ty_2Einteger_2Eint. \\
& (\forall V2q \in ty_2Einteger_2Eint. ((\exists V3r \in ty_2Einteger_2Eint. \\
& ((V0i = (ap (ap c_2Einteger_2Eint_add (ap (ap c_2Einteger_2Eint_mul \\
V2q) V1j)) V3r)) \wedge (p (ap (ap (ap (c_2Ebool_2ECOND 2) (ap (ap c_2Einteger_2Eint_lt \\
& V1j) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) (ap (ap c_2Ebool_2E_2F_5C \\
& (ap (ap c_2Einteger_2Eint_lt V1j) V3r)) (ap (ap c_2Einteger_2Eint_le \\
V3r) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)))) (ap (ap c_2Ebool_2E_2F_5C \\
& (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num \\
c_2Enum_2E0) V3r)) (ap (ap c_2Einteger_2Eint_lt V3r) V1j)))))) \Rightarrow \\
& ((ap (ap c_2Einteger_2Eint_div V0i) V1j) = V2q))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
A_27a A_27a) (c_2Emin_2E_3D A_27a)) (c_2Ecombin_2EI A_27a)) (\\
& c_2Ecombin_2EI A_27a)))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad (A_27b^{A_27a})\ (A_27d^{A_27c}))\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\
& \quad A_27a\ A_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27c \\
& \quad A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27a\ A_27d\ A_27c\ A_27b)\ V1abs1)\ V5rep2))))))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (\forall V6f \in (A_27d^{A_27c}). \\
& \quad ((\lambda V7x \in A_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x))))))))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0REL \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0REL)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& \quad A_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3f \in (2^{A_27b}).((p\ (\\
& \quad ap\ (c_2Ebool_2E_21\ A_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2ERespects\ A_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b\ 2)\ V1abs)\ (c_2Ecombin_2EI \\
& \quad 2))\ V3f))))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27a}).(\forall V4g \in \\
& \quad (2^{A_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad 2)\ V0R)\ (c_2Emin_2E_3D\ 2)\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (\\
& \quad ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0R))\ V4g))))))))) \\
& \hspace{15em} (54)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b^{A_27a}). \\
& \quad (\forall V7g \in (A_27b^{A_27a}).(\forall V8x \in A_27a).(\forall V9y \in \\
& \quad A_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))) \\
& \hspace{15em} (55)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0E \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1P \in (2^{A_27a}).((p\ (ap\ (c_2Equotient_2EEQUIV\ A_27a) \\
& \quad V0E)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E_21\ A_27a)\ V1P)))))) \\
& \hspace{15em} (56)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \hspace{10em} (57)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \hspace{10em} (58)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
& \hspace{15em} (59)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
& \hspace{15em} (60)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (61)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge (\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (66)$$

Theorem 1

$$\begin{aligned} & (\forall V0i \in ty_2Einteger_2Eint.(\forall V1j \in ty_2Einteger_2Eint. \\ & (\forall V2m \in ty_2Einteger_2Eint.((\exists V3q \in ty_2Einteger_2Eint. \\ & ((V0i = (ap (ap c_2Einteger_2Eint_add (ap (ap c_2Einteger_2Eint_mul \\ & V3q) V1j)) V2m)) \wedge (p (ap (ap (ap (c_2Ebool_2ECOND 2) (ap (ap c_2Einteger_2Eint_lt \\ & V1j) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) (ap (ap c_2Ebool_2E_2F_5C \\ & (ap (ap c_2Einteger_2Eint_lt V1j) V2m)) (ap (ap c_2Einteger_2Eint_le \\ & V2m) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)))) (ap (ap c_2Ebool_2E_2F_5C \\ & (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num \\ & c_2Enum_2E0) V2m)) (ap (ap c_2Einteger_2Eint_lt V2m) V1j)))))) \Rightarrow \\ & ((ap (ap c_2Einteger_2Eint_mod V0i) V1j) = V2m)))))) \end{aligned}$$