thm_2Einteger_2EINT__MUL__CALCULATE (TMNU3gQofJx2MhZTbNaC5X5VtvQyuuk3PRX)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{3}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{4}$$

Definition 6 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 7 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (ty$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (5)$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum \quad (6)$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)} \quad (7)$$

Definition 8 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 9 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint__

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (9)$$

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \quad (10)$$

Definition 10 *We define* c_2Einteger_2Eint__mul *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Intege$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x))\ V1y)))) \quad (12)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1y))))) \quad (13)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x)) = V0x)) \tag{14}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))))) \tag{15}$$

Theorem 1

$$((\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)))))\land((\forall V2x \in ty\_2Einteger\_2Eint.(\forall V3y \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V2x))\ V3y) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2x)\ V3y)))))\land((\forall V4x \in ty\_2Einteger\_2Eint.(\forall V5y \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V4x)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V5y)) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V4x)\ V5y)))))\land(\forall V6x \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V6x)) = V6x)))))$$