

thm_2Einteger_2EINT__MUL__RZERO (TMczrdgDfM3k2B5zsuJdSKiEmXLq83ys5NK)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V 0t \in 2. V 0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V 0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V 0t)) \text{c_2Ebool_2E_2F}))$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \tag{1}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A 0 A 1) \tag{2}$$

Let `c_2Einteger_2Etint_eq` : ι be given. Assume the following.

$$\text{c_2Einteger_2Etint_eq} \in ((2^{(\text{ty_2Epair_2Eprod } \text{ty_2Enum_2Enum } \text{ty_2Enum_2Enum})}) (\text{ty_2Epair_2Eprod } \text{ty_2Enum_2Enum})) \tag{3}$$

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Einteger_2Eint} \tag{4}$$

Let `c_2Einteger_2Eint__REP__CLASS` : ι be given. Assume the following.

$$\text{c_2Einteger_2Eint_REP_CLASS} \in ((2^{(\text{ty_2Epair_2Eprod } \text{ty_2Enum_2Enum } \text{ty_2Enum_2Enum})}) \text{ty_2Einteger_2Eint}) \tag{5}$$

Definition 7 We define $c_2Emin_2E.40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 8 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint$.(ap (c_2Emin_2E.40 (ty_2Einteger_2Eint_mul : ι be given. Assume the following.

$$c_2Einteger_2Eint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})} \quad (7)$$

Definition 9 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (9)$$

Definition 10 We define c_2Enum_2E0 to be (ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP).

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint)^{ty_2Enum_2Enum} \quad (10)$$

Definition 11 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint$. $\lambda V1T2 \in ty_2Einteger$

Definition 12 We define $c_2Ecombin_2EK$ to be $\lambda A.27a : \iota$. $\lambda A.27b : \iota$.($\lambda V0x \in A.27a$.($\lambda V1y \in A.27b$. $V0x$))

Definition 13 We define $c_2Ecombin_2ES$ to be $\lambda A.27a : \iota$. $\lambda A.27b : \iota$. $\lambda A.27c : \iota$.($\lambda V0f \in ((A.27c^{A.27b})^{A.27a})$

Definition 14 We define $c_2Ecombin_2EI$ to be $\lambda A.27a : \iota$.(ap (ap (c_2Ecombin_2ES A.27a (A.27a^{A.27a}) A

Definition 15 We define $c_2Equotient_2E.2D.2D.3E$ to be $\lambda A.27a : \iota$. $\lambda A.27b : \iota$. $\lambda A.27c : \iota$. $\lambda A.27d : \iota$. $\lambda V0j$

Definition 16 We define $c_2Equotient_2E.3D.3D.3E$ to be $\lambda A.27a : \iota$. $\lambda A.27b : \iota$. $\lambda V0R1 \in ((2^{A.27a})^{A.27b})$

Definition 17 We define $c_2Ebool_2E.2F.5C$ to be ($\lambda V0t1 \in 2$.($\lambda V1t2 \in 2$.(ap (c_2Ebool_2E.21 2) ($\lambda V2t \in$

Definition 18 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A.27a : \iota$. $\lambda A.27b : \iota$. $\lambda V0R \in ((2^{A.27a})^{A.27b})$.

Definition 19 We define $c_2Ecombin_2EW$ to be $\lambda A.27a : \iota$. $\lambda A.27b : \iota$.($\lambda V0f \in ((A.27b^{A.27a})^{A.27a})$).($\lambda V1x$

Definition 20 We define $c_2Equotient_2Erespects$ to be $\lambda A.27a : \iota$. $\lambda A.27b : \iota$.(c_2Ecombin_2EW A.27a A.27b

Definition 21 We define `c_2Ebool_2EIN` to be $\lambda A_{.27a} : \iota. (\lambda V0x \in A_{.27a}. (\lambda V1f \in (2^{A_{.27a}}). (ap\ V1f\ V0x)))$

Definition 22 We define `c_2Ebool_2ERES_FORALL` to be $\lambda A_{.27a} : \iota. (\lambda V0p \in (2^{A_{.27a}}). (\lambda V1m \in (2^{A_{.27a}}). ($

Definition 23 We define `c_2Equotient_2EEQUIV` to be $\lambda A_{.27a} : \iota. \lambda V0E \in ((2^{A_{.27a}})^{A_{.27a}}). (ap\ (c_2Ebool_2EIN$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\forall A_{.27a}. nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. ((V0x = V0x) \Leftrightarrow True)) \tag{13}$$

Assume the following.

$$\forall A_{.27a}. nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\forall V1y \in A_{.27a}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{14}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \tag{15}$$

Assume the following.

$$\forall A_{.27a}. nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. ((ap\ (c_2Ecombin_2EI\ A_{.27a})\ V0x) = V0x)) \tag{16}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). \\ & (\forall V1q \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). \\ & ((p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0p)\ V1q)) \Leftrightarrow ((ap\ c_2Einteger_2Etint_eq \\ & V0p) = (ap\ c_2Einteger_2Etint_eq\ V1q)))) \end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). \\ & (\forall V1q \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). \\ & ((V0p = V1q) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0p)\ V1q)))) \end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). \\ & (\forall V1y \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). \\ & ((ap\ (ap\ c_2Einteger_2Etint_mul\ V0x)\ V1y) = (ap\ (ap\ c_2Einteger_2Etint_mul \\ & V1y)\ V0x)))) \end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1x2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V2y1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V3y2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (((p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V2y1)\ V3y2))) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ (ap\ (ap\ c_2Einteger_2Etint_mul\ V0x1)\ V2y1))\ (ap\ (ap\ c_2Einteger_2Etint_mul\ V1x2)\ V3y2))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)\ ty_2Einteger_2Eint)\ c_2Einteger_2Etint_eq)\ c_2Einteger_2Eint_ABS)\ c_2Einteger_2Eint_REP))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap\ (ap\ c_2Einteger_2Eint_mul\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))\ V0x) = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27a)\ (c_2Emin_2E_3D\ A_27a))\ (c_2Ecombin_2EI\ A_27a))\ (c_2Ecombin_2EI\ A_27a)))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}). (\forall V1abs1 \in (A_27c)^{A_27a}). (\forall V2rep1 \in \\
& \quad (A_27a)^{A_27c}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c)\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}). (\forall V4abs2 \in \\
& \quad (A_27d)^{A_27b}). (\forall V5rep2 \in (A_27b)^{A_27d}). ((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ (A_27b)^{A_27a})\ (A_27d)^{A_27c})\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a\ A_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27a\ A_27d\ A_27c\ A_27b)\ V1abs1)\ V5rep2))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x \in A_27b.(\forall V4y \in \\
& A_27b.((V3x = V4y) \Leftrightarrow (p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3x))\ (ap\ V2rep\ V4y)))))))))) \\
& \hspace{15em} (25)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& A_27a.(\forall V5y1 \in A_27a.(\forall V6y2 \in A_27a.(((p\ (ap\ (ap\ V0R \\
& V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2)))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2)))))))))) \\
& \hspace{15em} (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (\forall V6f \in (A_27d^{A_27c}). \\
& ((\lambda V7x \in A_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x)))))))))) \\
& \hspace{15em} (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0REL \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0REL)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& A_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2)))))))))) \\
& \hspace{15em} (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_{.27a}})^{A_{.27a}}).(\forall V1abs \in (A_{.27b}^{A_{.27a}}). \\
& \quad (\forall V2rep \in (A_{.27a}^{A_{.27b}}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad \quad A_{.27a}\ A_{.27b})\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3f \in (2^{A_{.27b}}).((p\ (\\
& \quad ap\ (c_2Ebool_2E_21\ A_{.27b})\ V3f))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad \quad A_{.27a})\ (ap\ (c_2Equotient_2ERespects\ A_{.27a}\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad \quad (c_2Equotient_2E_2D_2D_3E\ A_{.27a}\ 2\ A_{.27b}\ 2)\ V1abs)\ (c_2Ecombin_2EI \\
& \quad \quad 2))\ V3f)))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_{.27a}})^{A_{.27a}}).(\forall V1abs \in (A_{.27b}^{A_{.27a}}). \\
& \quad (\forall V2rep \in (A_{.27a}^{A_{.27b}}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad \quad A_{.27a}\ A_{.27b})\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3f \in (2^{A_{.27a}}).(\forall V4g \in \\
& \quad (2^{A_{.27a}}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_{.27a} \\
& \quad 2)\ V0R)\ (c_2Emin_2E_3D\ 2)\ V3f)\ V4g))) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad \quad A_{.27a})\ (ap\ (c_2Equotient_2ERespects\ A_{.27a}\ 2)\ V0R))\ V3f))) \Leftrightarrow (p\ (\\
& \quad ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_{.27a})\ (ap\ (c_2Equotient_2ERespects \\
& \quad \quad A_{.27a}\ 2)\ V0R))\ V4g)))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}. \\
& \quad nonempty\ A_{.27c} \Rightarrow \forall A_{.27d}.nonempty\ A_{.27d} \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_{.27a}})^{A_{.27a}}).(\forall V1abs1 \in (A_{.27c}^{A_{.27a}}).(\forall V2rep1 \in \\
& \quad (A_{.27a}^{A_{.27c}}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_{.27a}\ A_{.27c}) \\
& \quad \quad V0R1)\ V1abs1)\ V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A_{.27b}})^{A_{.27b}}).(\forall V4abs2 \in \\
& \quad (A_{.27d}^{A_{.27b}}).(\forall V5rep2 \in (A_{.27b}^{A_{.27d}}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad \quad A_{.27b}\ A_{.27d})\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (\forall V6f \in (A_{.27b}^{A_{.27a}}). \\
& \quad (\forall V7g \in (A_{.27b}^{A_{.27a}}).(\forall V8x \in A_{.27a}).(\forall V9y \in \\
& \quad A_{.27a}.(((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_{.27a} \\
& \quad A_{.27b})\ V0R1)\ V3R2)\ V6f)\ V7g))) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y)))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0E \in ((2^{A_{.27a}})^{A_{.27a}}). \\
& \quad (\forall V1P \in (2^{A_{.27a}}).((p\ (ap\ (c_2Equotient_2EEQUIV\ A_{.27a}) \\
& \quad V0E))) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_{.27a})\ (ap\ (c_2Equotient_2ERespects \\
& \quad \quad A_{.27a}\ 2)\ V0E))\ V1P))) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E_21\ A_{.27a})\ V1P))))))
\end{aligned} \tag{32}$$

Theorem 1

$$(\forall V0x \in ty_2Einteger_2Eint.((ap\ (ap\ c_2Einteger_2Eint_mul \\
V0x)\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) = (ap\ c_2Einteger_2Eint_of_num \\
c_2Enum_2E0)))$$