

thm_2Einteger_2EINT__NEG__EQ0
(TMaE6FtiQr9isXBHAt46R3ze2gWZQzBYTqM)

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Definition 1 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p \Rightarrow Q)$ of type ι .

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty \ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 2 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Enum_2E0 to be $(ap \ c_2Enum_2EABS_num \ c_2Enum_2EZERO_REP)$.

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty \ ty_2Einteger_2Eint \tag{4}$$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \tag{5}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow \forall A1.nonempty \ A1 \Rightarrow nonempty \ (ty_2Epair_2Eprod \ A0 \ A1) \tag{6}$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod \ ty_2Enum_2Enum \ ty_2Enum_2Enum)}) \ ty_2Einteger_2Eint) \tag{7}$$

Definition 4 We define $c_2Emin_2E.40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 5 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E.3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E.21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E.3D (2^{A-27a}))$

Definition 7 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E.40 (ty$

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum) (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)) \quad (8)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)) \quad (9)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})} \quad (10)$$

Definition 8 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$

Definition 9 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint$

Let $c_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum) (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)) (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)) \quad (11)$$

Definition 10 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (12)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.((ap (ap c_2Einteger_2Eint_add V0x) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) = V0x)) \quad (13)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg V0x) V0x) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \quad (14)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add \\
 & V0x) (ap c_2Einteger_2Eint_neg V0x)) = (ap c_2Einteger_2Eint_of_num \\
 & \quad c_2Enum_2E0)))
 \end{aligned}
 \tag{15}$$

Theorem 1

$$\begin{aligned}
 & (\forall V0x \in ty_2Einteger_2Eint. (((ap c_2Einteger_2Eint_neg \\
 & V0x) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \Leftrightarrow (V0x = (ap \\
 & \quad c_2Einteger_2Eint_of_num c_2Enum_2E0))))
 \end{aligned}$$