

thm\_2Einteger\_2EINT\_\_NEG\_\_LE0  
(TMVzrw8aeSqgcj1SoZwH5XSCsCN1upzcP6Q)

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**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E21$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E21$  to be  $(ap (c\_2Ebool\_2E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E3D\_3D\_3E V0t) c\_2Ebool\_2E21))$

Let  $ty\_2Eenum\_2Eenum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eenum\_2Eenum \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{3}$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Eenum\_2Eenum\ ty\_2Eenum\_2Eenum)})\ ty\_2Einteger\_2Eint) \tag{4}$$

**Definition 7** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E40 (ty\_2Einteger\_2Eint\_REP\_CLASS V0a)))$

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)) \quad (5)$$

**Definition 9** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint$ .

**Definition 10** We define  $c\_2Einteger\_2Eint\_le$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2Eint$ .

Let  $c\_2Einteger\_2Etint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (6)$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)) \quad (7)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})} \quad (8)$$

**Definition 11** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$ .

**Definition 12** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap\ c\_2Einteger\_2Eint$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (9)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (10)$$

**Definition 13** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint)^{ty\_2Enum\_2Enum} \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt \\ & (ap\ c\_2Einteger\_2Eint\_of\_num\ c\_2Enum\_2E0))\ (ap\ c\_2Einteger\_2Eint\_neg \\ & V0x))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ V0x)\ (ap\ c\_2Einteger\_2Eint\_of\_num \\ & c\_2Enum\_2E0)))))) \end{aligned} \quad (14)$$

**Theorem 1**

$$(\forall V0x \in ty\_2Einteger\_2Eint. ((p (ap (ap c\_2Einteger\_2Eint\_le (ap c\_2Einteger\_2Eint\_neg V0x)) (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0))) \Leftrightarrow (p (ap (ap c\_2Einteger\_2Eint\_le (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) V0x))))$$