

thm_2Einteger_2EINT__NEG__MUL2 (TMRy8g4dmFFusCZ8AWQEXStU2Ho3us2g7J7)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Eenum_2E_enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2E_enum \tag{1}$$

Let $ty_2Epair_2E_eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2E_eprod\ A0\ A1) \tag{2}$$

Let $ty_2Einteger_2E_eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2E_eint \tag{3}$$

Let $c_2Einteger_2E_eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2E_eint_REP_CLASS \in ((2^{(ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum)})^{ty_2Einteger_2E_eint}) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p)$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define $c_2Einteger_2E_eint_REP$ to be $\lambda V0a \in ty_2Einteger_2E_eint.(ap (c_2Emin_2E_40 (ty$

Let $c_2Einteger_2E_etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2E_etint_mul \in (((ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum)^{ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum})^{ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum}) \tag{5}$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})(ty_2Epair_2Eprod\ ty_2Enum_2Enum)) \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (7)$$

Definition 6 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 7 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (8)$$

Definition 8 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint_neg$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. \\ & ((ap\ c_2Einteger_2Eint_neg\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x) \\ & V1y)) = (ap\ (ap\ c_2Einteger_2Eint_mul\ (ap\ c_2Einteger_2Eint_neg\ V0x))\ V1y)))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. \\ & ((ap\ c_2Einteger_2Eint_neg\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x) \\ & V1y)) = (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x)\ (ap\ c_2Einteger_2Eint_neg\ V1y)))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Einteger_2Eint.((ap\ c_2Einteger_2Eint_neg \\ & (ap\ c_2Einteger_2Eint_neg\ V0x)) = V0x)) \end{aligned} \quad (15)$$

Theorem 1

$$\begin{aligned} & (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\ & ((ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_neg V0x)) \\ & (ap c_2Einteger_2Eint_neg V1y)) = (ap (ap c_2Einteger_2Eint_mul \\ & V0x) V1y)))) \end{aligned}$$