

thm_2Einteger_2EINT__QUOT (TMJFQFHpb- vZUhbUHcfWmwmAfgSqXPXQQNMb)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (2)$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (3)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (4)$$

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p\ (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p\ x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2ET$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

Definition 5 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E_40\ (ty_2Einteger_2Eint_REP_CLASS\ a)))$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (5)$$

Let $c_Einteger_Etint_eq : \iota$ be given. Assume the following.

$$c_Einteger_Etint_eq \in ((2^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)}) (ty_Epair_Eprod\ ty_Eenum_Eenum)) \quad (6)$$

Let $c_Einteger_Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Einteger_Eint_ABS_CLASS \in (ty_Einteger_Eint)^{(2^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)})} \quad (7)$$

Definition 6 We define $c_Einteger_Eint_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)$

Definition 7 We define $c_Einteger_Eint_neg$ to be $\lambda V0T1 \in ty_Einteger_Eint.(ap\ c_Einteger_Eint_neg)$

Let $c_Einteger_Eint_of_num : \iota$ be given. Assume the following.

$$c_Einteger_Eint_of_num \in (ty_Einteger_Eint)^{ty_Eenum_Eenum} \quad (8)$$

Definition 8 We define $c_Einteger_Eenum$ to be $\lambda V0i \in ty_Einteger_Eint.(ap\ (c_Emin_E40\ ty_Eenum_Eenum))$

Let $c_Earithmetic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_EDIV \in ((ty_Eenum_Eenum)^{ty_Eenum_Eenum})^{ty_Eenum_Eenum} \quad (9)$$

Let $c_Einteger_Etint_lt : \iota$ be given. Assume the following.

$$c_Einteger_Etint_lt \in ((2^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)}) (ty_Epair_Eprod\ ty_Eenum_Eenum)) \quad (10)$$

Definition 9 We define $c_Einteger_Eint_lt$ to be $\lambda V0T1 \in ty_Einteger_Eint.\lambda V1T2 \in ty_Einteger_Eint.$

Definition 10 We define c_Ebool_EF to be $(ap\ (c_Ebool_E21\ 2))\ (\lambda V0t \in 2.V0t)$.

Definition 11 We define $c_Emin_E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 12 We define c_Ebool_E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_Emin_E3D_3D_3E\ V0t)\ c_Ebool_E7E))$

Definition 13 We define $c_Einteger_Eint_le$ to be $\lambda V0x \in ty_Einteger_Eint.\lambda V1y \in ty_Einteger_Eint.$

Definition 14 We define $c_Ebool_E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E21\ 2))\ (\lambda V2t \in 2.$

Definition 15 We define c_Ebool_ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda V3t3 \in A_27a.$

Let $c_Einteger_Eint_quot : \iota$ be given. Assume the following.

$$c_Einteger_Eint_quot \in ((ty_Einteger_Eint)^{ty_Einteger_Eint})^{ty_Einteger_Eint} \quad (11)$$

Let $c_Earithmetic_EEVEN : \iota$ be given. Assume the following.

$$c_Earithmetic_EEVEN \in (2^{ty_Eenum_Eenum}) \quad (12)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (14)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (15)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (16)$$

Definition 16 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 18 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 19 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 21 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (17)$$

Definition 22 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 23 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_2Ebool_2E$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Definition 24 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 25 We define $c_2\text{Earithmetic_2ENUMERAL}$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2\text{Earithmetic_2E_2B} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_2E_2B} \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (21)$$

Definition 26 We define $c_2\text{Earithmetic_2EBIT2}$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2\text{Earithmetic_2E_2B} V0n))$

Definition 27 We define $c_2\text{Earithmetic_2EBIT1}$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2\text{Earithmetic_2E_2B} V0n))$

Definition 28 We define $c_2\text{Earithmetic_2EZERO}$ to be c_2Enum_2E0 .

Definition 29 We define $c_2\text{Earithmetic_2E_3C_3D}$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2\text{Earithmetic_2E_3C_3D} c_2Enum_2E0) V0n))) \quad (22)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\neg(p (ap (ap c_2\text{Eprim_rec_2E_3C} V0m) V1n))) \Leftrightarrow (p (ap (ap c_2\text{Earithmetic_2E_3C_3D} V1n) V0m))))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2\text{Earithmetic_2E_2A} c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2\text{Earithmetic_2E_2A} V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2\text{Earithmetic_2E_2A} (ap c_2\text{Earithmetic_2ENUMERAL} \\ & (ap c_2\text{Earithmetic_2EBIT1} c_2\text{Earithmetic_2EZERO})) V0m) = V0m) \wedge \\ & (((ap (ap c_2\text{Earithmetic_2E_2A} V0m) (ap c_2\text{Earithmetic_2ENUMERAL} \\ & (ap c_2\text{Earithmetic_2EBIT1} c_2\text{Earithmetic_2EZERO})) = V0m) \wedge \\ & ((ap (ap c_2\text{Earithmetic_2E_2A} (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\ & (ap c_2\text{Earithmetic_2E_2B} (ap (ap c_2\text{Earithmetic_2E_2A} V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c_2\text{Earithmetic_2E_2A} V0m) (ap c_2Enum_2ESUC V1n)) = \\ & (ap (ap c_2\text{Earithmetic_2E_2B} V0m) (ap (ap c_2\text{Earithmetic_2E_2A} \\ & V0m) V1n)))))))))) \quad (24) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.(((p (ap (ap c_2\text{Earithmetic_2E_3C_3D} \\ & V0m) V1n)) \wedge (p (ap (ap c_2\text{Earithmetic_2E_3C_3D} V1n) V2p))) \Rightarrow (p (\\ & ap (ap c_2\text{Earithmetic_2E_3C_3D} V0m) V2p)))))) \quad (25) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \\
& \hspace{15em} (26)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Enum_2ESUC V1n)) V0m)))))) \\
& \hspace{15em} (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg (V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V1n)) V0m)))))) \\
& \hspace{15em} (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
& c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) V0n))) \\
& \hspace{15em} (29)
\end{aligned}$$

Assume the following.

$$\text{True} \hspace{15em} (30)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \\
& \hspace{15em} (31)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \hspace{15em} (32)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
& A_27a. (p V0t)) \Leftrightarrow (p V0t))) \\
& \hspace{15em} (33)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& (p V0t)) \Leftrightarrow (p V0t)))))) \\
& \hspace{15em} (34)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (35)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg (p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \ V1t2) = V0t1) \wedge ((ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \ V0t1) \ V1t2) = V1t2)))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B)))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \Rightarrow (p \ V1B)) \Leftrightarrow ((\neg(p \ V0A)) \vee (p \ V1B)))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Leftrightarrow ((p \ V0t) \Leftrightarrow False))) \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (44)$$

Assume the following.

$$2.((\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_{27})) \wedge ((p \ V1x_{27}) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_{27})))))) \Rightarrow ((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_{27}) \Rightarrow (p \ V3y_{27})))))) \Rightarrow \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty \ A_{27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_{27a}.(\forall V3x_{27} \in A_{27a}.(\forall V4y \in A_{27a}. \\ & (\forall V5y_{27} \in A_{27a}.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge (((p \ V1Q) \Rightarrow (V2x = V3x_{27})) \wedge \\ & ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_{27})))))) \Rightarrow ((ap \ (ap \ (ap \ (c_{2Ebool_2ECOND} \ A_{27a}) \\ & V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c_{2Ebool_2ECOND} \ A_{27a}) \ V1Q) \ V3x_{27}) \\ & V5y_{27})))))))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (p \ (ap \ (ap \ c_{2Einteger_2Eint_le} \ (ap \ c_{2Einteger_2Eint_of_num} \\ & V0m)) \ (ap \ c_{2Einteger_2Eint_of_num} \ V1n))) \Leftrightarrow (p \ (ap \ (ap \ c_{2Earithmetic_2E_3C_3D} \\ & V0m) \ V1n)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (p \ (ap \ (ap \ c_{2Einteger_2Eint_lt} \ (ap \ c_{2Einteger_2Eint_of_num} \\ & V0m)) \ (ap \ c_{2Einteger_2Eint_of_num} \ V1n))) \Leftrightarrow (p \ (ap \ (ap \ c_{2Eprim_rec_2E_3C} \\ & V0m) \ V1n)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & ((ap \ c_{2Einteger_2Eint_of_num} \ V0m) = (ap \ c_{2Einteger_2Eint_of_num} \\ & V1n)) \Leftrightarrow (V0m = V1n)))) \end{aligned} \quad (49)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap \ c_{2Einteger_2ENUM} \ (ap \ c_{2Einteger_2Eint_of_num} \ V0n)) = V0n)) \quad (50)$$

Assume the following.

$$\begin{aligned}
& (\forall V0i \in ty_2Einteger_2Eint. (\forall V1j \in ty_2Einteger_2Eint. \\
& ((\neg(V1j = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))) \Rightarrow ((ap \\
& (ap\ c_2Einteger_2Eint_quot\ V0i)\ V1j) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& ty_2Einteger_2Eint)\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))\ V1j))\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Einteger_2Eint) \\
& (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))\ V0i))\ (ap\ c_2Einteger_2Eint_of_num\ (ap\ (ap\ c_2Earithmetic_2EDIV \\
& (ap\ c_2Einteger_2Enum\ V0i))\ (ap\ c_2Einteger_2Enum\ V1j))))\ (ap \\
& c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num\ (ap\ (\\
& ap\ c_2Earithmetic_2EDIV\ (ap\ c_2Einteger_2Enum\ (ap\ c_2Einteger_2Eint_neg \\
& V0i)))\ (ap\ c_2Einteger_2Enum\ V1j))))))\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& ty_2Einteger_2Eint)\ (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))\ V0i))\ (ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num \\
& (ap\ (ap\ c_2Earithmetic_2EDIV\ (ap\ c_2Einteger_2Enum\ V0i))\ (ap\ c_2Einteger_2Enum \\
& (ap\ c_2Einteger_2Eint_neg\ V1j))))))\ (ap\ c_2Einteger_2Eint_of_num \\
& (ap\ (ap\ c_2Earithmetic_2EDIV\ (ap\ c_2Einteger_2Enum\ (ap\ c_2Einteger_2Eint_neg \\
& V0i)))\ (ap\ c_2Einteger_2Enum\ (ap\ c_2Einteger_2Eint_neg\ V1j)))))))))) \\
& (51)
\end{aligned}$$

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2B \\
& \quad (ap c_2Earithmic_2ENUMERAL V2n)) (ap c_2Earithmic_2ENUMERAL \\
& \quad V3m))) = (ap c_2Earithmic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2A (\\
& \quad ap c_2Earithmic_2ENUMERAL V6n)) (ap c_2Earithmic_2ENUMERAL \\
& \quad V7m))) = (ap c_2Earithmic_2ENUMERAL (ap (ap c_2Earithmic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2D \\
& \quad (ap c_2Earithmic_2ENUMERAL V10n)) (ap c_2Earithmic_2ENUMERAL \\
& \quad V11m))) = (ap c_2Earithmic_2ENUMERAL (ap (ap c_2Earithmic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmic_2EEXP c_2Enum_2E0) (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap c_2Earithmic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmic_2EEXP (ap c_2Earithmic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmic_2ENUMERAL V16m))) = (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmic_2EEXP V15n) V16m)))))) \wedge (((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 \\
& \quad c_2Earithmic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum.(\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmic_2ENUMERAL V17n)) = (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmic_2ENUMERAL V18n)) = (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmic_2EZERO)))))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmic_2E_3E (ap c_2Earithmic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmic_2E_3E (ap c_2Earithmic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL \\
& \quad V32n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D c_2Earithmic_2EZERO) \\
& \quad V32n)))))) \wedge ((\forall V33n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V33n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V33n)) c_2Enum_2E0)))))) \wedge ((\forall V34n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V34n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V34n)) c_2Enum_2E0)))))) \wedge ((\forall V35n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V35n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V35n)) c_2Enum_2E0)))))) \wedge ((\forall V36n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V36n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V36n)) c_2Enum_2E0)))))) \wedge ((\forall V37n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V37n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V37n)) c_2Enum_2E0)))))) \wedge ((\forall V38n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V38n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V38n)) c_2Enum_2E0)))))) \wedge ((\forall V39n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V39n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V39n)) c_2Enum_2E0)))))) \wedge ((\forall V40n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V40n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V40n)) c_2Enum_2E0)))))) \wedge ((\forall V41n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V41n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V41n)) c_2Enum_2E0)))))) \wedge ((\forall V42n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V42n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V42n)) c_2Enum_2E0)))))) \wedge ((\forall V43n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V43n)) c_2Enum_2E0)) \Leftrightarrow (p$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{54}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{57}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \\
& \hspace{25em} (61)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))) \\
& \hspace{25em} (62)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0p \in ty_2Enum_2Enum. (\forall V1q \in ty_2Enum_2Enum. (\\
& (\neg(V1q = c_2Enum_2E0)) \Rightarrow ((ap \ (ap \ c_2Einteger_2Eint_quot \ (ap \ c_2Einteger_2Eint_of_num \\
& \ V0p)) \ (ap \ c_2Einteger_2Eint_of_num \ V1q)) = (ap \ c_2Einteger_2Eint_of_num \\
& \ (ap \ (ap \ c_2Earithmetric_2EDIV \ V0p) \ V1q))))))
\end{aligned}$$