

thm_2Einteger_2EINT__REMQUOT (TMLD5nn7MEmv6uZZiNvnCvtu8bVT3X82cej)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{3}$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \tag{4}$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (5)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (6)$$

Definition 9 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E40\ (ty_2Einteger_2Eint\ a)))$

Let $c_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Einteger_2Eint})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (7)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (8)$$

Definition 10 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Let $c_2Einteger_2Eint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (9)$$

Definition 11 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 12 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E\ t))$

Definition 13 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$

Definition 14 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 15 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ (ty_2Einteger_2Eint\ P)))))$

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Einteger_2Eint}) \quad (10)$$

Definition 16 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint_neg\ T1)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (12)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (13)$$

Definition 18 We define $c_2Einteger_2EABS$ to be $\lambda V0n \in ty_2Einteger_2Eint.(ap\ (ap\ (ap\ (c_2Ebool_2E0$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Let $c_2Einteger_2Eint_quot : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_quot \in ((ty_2Einteger_2Eint^{ty_2Einteger_2Eint})^{ty_2Einteger_2Eint}) \quad (15)$$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (16)$$

Definition 19 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 20 We define $c_2Einteger_2Eint_sub$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$

Let $c_2Einteger_2Eint_rem : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_rem \in ((ty_2Einteger_2Eint^{ty_2Einteger_2Eint})^{ty_2Einteger_2Eint}) \quad (17)$$

Definition 21 We define $c_2Emarker_2EAbbrev$ to be $\lambda V0x \in 2.V0x$.

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (18)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (21)$$

Definition 22 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 23 We define $c_2Eprim_rec_2E.3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 24 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$.

Definition 25 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2)))$.

Definition 26 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$.

Definition 27 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E_21) 2) 2) 2)$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (22)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (23)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (24)$$

Definition 28 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 29 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (25)$$

Definition 30 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B) 2)$.

Definition 31 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B) 2)$.

Definition 32 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 33 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$.

Definition 34 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$.

Definition 35 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$.

Definition 36 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES) A_27a) (A_27a^{A_27a}))$.

Definition 37 We define $c_2Equotient_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$.

Definition 38 We define $c_2Equotient_2E_3D_3D_3D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R1 \in ((2^{A_27a})^{A_27b})$.

Definition 39 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27b})$.

Definition 40 We define $c_2Ecombin_2EW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in ((A_27b^{A_27a})^{A_27a}).(\lambda V1x$

Definition 41 We define $c_2Equotient_2Erespects$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(c_2Ecombin_2EW A_27a A_27b$

Definition 42 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Definition 43 We define $c_2Ebool_2ERES_FORALL$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).$

Definition 44 We define $c_2Equotient_2EEQUIV$ to be $\lambda A_27a : \iota.\lambda V0E \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n)))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))))))) \\ & \hspace{15em} (26) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \\ & \hspace{15em} (27) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \\ & \hspace{15em} (28) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.((\neg(V0n = c_2Enum_2E0)) \Leftrightarrow (p (ap (\\ & ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)))) \\ & \hspace{15em} (29) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\ & (ap c_2Enum_2ESUC V0m)) V1n)))))) \\ & \hspace{15em} (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D \\ & c_2Enum_2E0) V0n))) \\ & \hspace{15em} (31) \end{aligned}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))) \quad (32)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (33)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0)) \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\ & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\ & (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\ & (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\ & V0m) V1n)))))))))) \quad (35) \end{aligned}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A V1n) V0m)))) \quad (36)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C (ap (ap c_2Earithmetic_2E_2B V0m) V2p)) (ap (ap c_2Earithmetic_2E_2B V1n) V2p))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)))))) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \quad (38)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0m) V1n)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V1n) V2p))) \Rightarrow (p (ap (ap \\
& c_2Eprim_rec_2E_3C V0m) V2p))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap (ap c_2Earithmic_2E_2B V0m) V1n)) (ap (ap c_2Earithmic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg (p (ap (ap c_2Earithmic_2E_3C_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap c_2Enum_2ESUC V1n)) V0m))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg (V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V1n)) V0m))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
& c_2Earithmic_2E_2B (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 \\
& c_2Earithmic_2EZERO))) V0n)))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V0n)) \Rightarrow (\forall V1k \in ty_2Enum_2Enum. ((V1k = (ap (\\
& ap c_2Earithmic_2E_2B (ap (ap c_2Earithmic_2E_2A (ap (ap c_2Earithmic_2EDIV \\
& V1k) V0n)) V0n)) (ap (ap c_2Earithmic_2EMOD V1k) V0n))) \wedge (p (ap \\
& (ap c_2Eprim_rec_2E_3C (ap (ap c_2Earithmic_2EMOD V1k) V0n)) \\
& V0n))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V0n)) \Rightarrow ((ap (ap c_2Earithmic_2EDIV c_2Enum_2E0) \\
& V0n) = c_2Enum_2E0)))
\end{aligned} \tag{45}$$

Assume the following.

$$True \quad (46)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (48)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \quad (49)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (50)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (51)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (52)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (53)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (55)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (56)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (57)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (58)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (59)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (60)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\forall V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (61)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a. (p\ (ap\ V1Q\ V4x)))))) \quad (62)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\exists V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A_27a. (p\ (ap\ V1Q\ V4x)))))) \quad (63)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (65)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (66)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)))) \quad (68)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow \text{False}) \Leftrightarrow ((p V0t) \Leftrightarrow \text{False}))) \quad (69)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (70)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2)))))) \quad (71)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((\text{ap } (\text{ap } (\text{ap } (\text{c.2Ebool_2ECOND } A_27a) \\ V0P) V2x) V4y) = (\text{ap } (\text{ap } (\text{ap } (\text{c.2Ebool_2ECOND } A_27a) V1Q) V3x_27) \\ V5y_27)))))))))) \quad (72) \end{aligned}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((\text{ap } (\text{c.2Ecombin_2EI } A_27a) V0x) = V0x)) \quad (73)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in (\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}). \\ (\forall V1q \in (\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}). \\ ((p (\text{ap } (\text{ap } \text{c.2Einteger_2Etint_eq } V0p) V1q)) \Leftrightarrow ((\text{ap } \text{c.2Einteger_2Etint_eq} \\ V0p) = (\text{ap } \text{c.2Einteger_2Etint_eq } V1q)))) \quad (74) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1q \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \quad (75) \\
& ((V0p = V1q) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0p)\ V1q))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1y \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V2z \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& ((ap\ (ap\ c_2Einteger_2Etint_add\ V0x)\ (ap\ (ap\ c_2Einteger_2Etint_add\ V1y)\ V2z))) = (ap\ (ap\ c_2Einteger_2Etint_add\ (ap\ (ap\ c_2Einteger_2Etint_add\ V0x)\ V1y))\ V2z)))) \quad (76)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V1x2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V2y1 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (\forall V3y2 \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (((p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V2y1)\ V3y2)))) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ (ap\ (ap\ c_2Einteger_2Etint_add\ V0x1)\ V2y1))\ (ap\ (ap\ c_2Einteger_2Etint_add\ V1x2)\ V3y2)))))) \quad (77)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)\ ty_2Einteger_2Eint)\ c_2Einteger_2Etint_eq)\ c_2Einteger_2Eint_ABS)\ c_2Einteger_2Eint_REP)) \quad (78)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Einteger_2Eint. (\forall V1x \in ty_2Einteger_2Eint. \\
& ((ap\ (ap\ c_2Einteger_2Eint_add\ V1x)\ V0y) = (ap\ (ap\ c_2Einteger_2Eint_add\ V0y)\ V1x)))) \quad (79)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)\ V0x) = V0x)) \quad (80)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap\ c_2Einteger_2Eint_neg\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x)\ V1y)) = (ap\ (ap\ c_2Einteger_2Eint_mul\ (ap\ c_2Einteger_2Eint_neg\ V0x)\ V1y)))) \quad (81)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap\ c_2Einteger_2Eint_neg\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x) \\
& V1y)) = (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x)\ (ap\ c_2Einteger_2Eint_neg \\
& V1y))))))
\end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap\ c_2Einteger_2Eint_neg \\
& (ap\ c_2Einteger_2Eint_neg\ V0x)) = V0x))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\
& (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))\ (ap\ c_2Einteger_2Eint_neg \\
& V0x))) \Leftrightarrow (p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ V0x)\ (ap\ c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& ((ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p\ (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ c_2Einteger_2Eint_of_num \\
& V0m))\ (ap\ c_2Einteger_2Eint_of_num\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0m)\ V1n))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_of_num \\
& V0m))\ (ap\ c_2Einteger_2Eint_of_num\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& V0m)\ V1n))))))
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Einteger_2Eint_of_num\ V0m) = (ap\ c_2Einteger_2Eint_of_num \\
& V1n))) \Leftrightarrow (V0m = V1n))))
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ c_2Einteger_2Eint_of_num \\
& V0m))\ (ap\ c_2Einteger_2Eint_of_num\ V1n)) = (ap\ c_2Einteger_2Eint_of_num \\
& (ap\ (ap\ c_2Earithmic_2E_2B\ V0m)\ V1n))))))
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (ap (ap c_2Integer_2Eint_mul (ap c_2Integer_2Eint_of_num \\
V0m)) (ap c_2Integer_2Eint_of_num V1n)) = (ap c_2Integer_2Eint_of_num \\
& \quad (ap (ap c_2Arithmetic_2E_2A V0m) V1n))))))
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Integer_2Eint. (\forall V1y \in ty_2Integer_2Eint. \\
& \quad (\forall V2z \in ty_2Integer_2Eint. ((p (ap (ap c_2Integer_2Eint_lt \\
(ap (ap c_2Integer_2Eint_sub V0x) V1y)) V2z))) \Leftrightarrow (p (ap (ap c_2Integer_2Eint_lt \\
& \quad V0x) (ap (ap c_2Integer_2Eint_add V2z) V1y)))))))
\end{aligned} \tag{91}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Integer_2Eint. (\forall V1y \in ty_2Integer_2Eint. \\
& \quad (\forall V2z \in ty_2Integer_2Eint. ((p (ap (ap c_2Integer_2Eint_lt \\
V0x) (ap (ap c_2Integer_2Eint_sub V1y) V2z))) \Leftrightarrow (p (ap (ap c_2Integer_2Eint_lt \\
& \quad (ap (ap c_2Integer_2Eint_add V0x) V2z)) V1y))))))
\end{aligned} \tag{92}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Integer_2Eint. (\forall V1y \in ty_2Integer_2Eint. \\
& \quad (\forall V2z \in ty_2Integer_2Eint. ((p (ap (ap c_2Integer_2Eint_le \\
V0x) (ap (ap c_2Integer_2Eint_sub V1y) V2z))) \Leftrightarrow (p (ap (ap c_2Integer_2Eint_le \\
& \quad (ap (ap c_2Integer_2Eint_add V0x) V2z)) V1y))))))
\end{aligned} \tag{93}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Integer_2Eint. (\forall V1y \in ty_2Integer_2Eint. \\
& \quad (\forall V2z \in ty_2Integer_2Eint. ((p (ap (ap c_2Integer_2Eint_le \\
(ap (ap c_2Integer_2Eint_sub V0x) V1y)) V2z))) \Leftrightarrow (p (ap (ap c_2Integer_2Eint_le \\
& \quad V0x) (ap (ap c_2Integer_2Eint_add V2z) V1y))))))
\end{aligned} \tag{94}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Integer_2Eint. (\forall V1y \in ty_2Integer_2Eint. \\
& \quad (\forall V2z \in ty_2Integer_2Eint. ((V0x = (ap (ap c_2Integer_2Eint_sub \\
V1y) V2z)) \Leftrightarrow ((ap (ap c_2Integer_2Eint_add V0x) V2z) = V1y))))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Integer_2Eint. ((\exists V1n \in ty_2Enum_2Enum. \\
((V0p = (ap c_2Integer_2Eint_of_num V1n)) \wedge (\neg (V1n = c_2Enum_2E0)))) \vee \\
& ((\exists V2n \in ty_2Enum_2Enum. (V0p = (ap c_2Integer_2Eint_neg \\
(ap c_2Integer_2Eint_of_num V2n))) \wedge (\neg (V2n = c_2Enum_2E0)))) \vee \\
& (V0p = (ap c_2Integer_2Eint_of_num c_2Enum_2E0))))))
\end{aligned} \tag{96}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap\ c_2Einteger_2EABS\ (ap\ c_2Einteger_2Eint_of_num\ V0n)) = (ap\ c_2Einteger_2Eint_of_num\ V0n))) \quad (97)$$

Assume the following.

$$(\forall V0p \in ty_2Einteger_2Eint.((ap\ c_2Einteger_2EABS\ (ap\ c_2Einteger_2Eint_neg\ V0p)) = (ap\ c_2Einteger_2EABS\ V0p))) \quad (98)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in ty_2Einteger_2Eint.(\forall V1q \in ty_2Einteger_2Eint. \\ & ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2EABS\ V0p)) \\ V1q)) \Leftrightarrow ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ V0p)\ V1q)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\ (ap\ c_2Einteger_2Eint_neg\ V1q))\ V0p)))) \wedge ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\ V1q)\ (ap\ c_2Einteger_2EABS\ V0p))) \Leftrightarrow ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\ V1q)\ V0p)) \vee (p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ V0p)\ (ap\ c_2Einteger_2Eint_neg \\ V1q)))))) \wedge ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_neg \\ (ap\ c_2Einteger_2EABS\ V0p))\ V1q)) \Leftrightarrow ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\ (ap\ c_2Einteger_2Eint_neg\ V1q))\ V0p)) \vee (p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\ V0p)\ V1q)))) \wedge ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ V1q)\ (ap\ c_2Einteger_2Eint_neg \\ (ap\ c_2Einteger_2EABS\ V0p)))) \Leftrightarrow ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\ V0p)\ (ap\ c_2Einteger_2Eint_neg\ V1q))) \wedge (p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\ V1q)\ V0p)))))))))) \end{aligned} \quad (99)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in ty_2Enum_2Enum.(\forall V1q \in ty_2Enum_2Enum.(\\ (\neg(V1q = c_2Enum_2E0)) \Rightarrow ((ap\ (ap\ c_2Einteger_2Eint_quot\ (ap\ c_2Einteger_2Eint_of_num \\ V0p))\ (ap\ c_2Einteger_2Eint_of_num\ V1q)) = (ap\ c_2Einteger_2Eint_of_num \\ (ap\ (ap\ c_2Earithmetric_2EDIV\ V0p)\ V1q)))))) \end{aligned} \quad (100)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in ty_2Einteger_2Eint.(\forall V1q \in ty_2Einteger_2Eint. \\ ((\neg(V1q = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))) \Rightarrow (((\\ ap\ (ap\ c_2Einteger_2Eint_quot\ (ap\ c_2Einteger_2Eint_neg\ V0p)) \\ V1q) = (ap\ c_2Einteger_2Eint_neg\ (ap\ (ap\ c_2Einteger_2Eint_quot \\ V0p)\ V1q))) \wedge ((ap\ (ap\ c_2Einteger_2Eint_quot\ V0p)\ (ap\ c_2Einteger_2Eint_neg \\ V1q)) = (ap\ c_2Einteger_2Eint_neg\ (ap\ (ap\ c_2Einteger_2Eint_quot \\ V0p)\ V1q)))))) \end{aligned} \quad (101)$$

Assume the following.

$$\begin{aligned}
& (\forall V0i \in ty_2Einteger_2Eint. (\forall V1j \in ty_2Einteger_2Eint. \\
& ((\neg(V1j = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))) \Rightarrow ((ap \\
& (ap\ c_2Einteger_2Eint_rem\ V0i)\ V1j) = (ap\ (ap\ c_2Einteger_2Eint_sub \\
& V0i)\ (ap\ (ap\ c_2Einteger_2Eint_mul\ (ap\ (ap\ c_2Einteger_2Eint_quot \\
& V0i)\ V1j))\ V1j))))))
\end{aligned}
\tag{102}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m)))))))))))))
\end{aligned} \tag{104}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg (p (ap (ap (ap c_2Eprim_rec_2E_3C V0n) c_2Enum_2E0)))) \tag{105}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (p (ap (ap (ap (c_2Equotient_2EQUOTIENT A_27a A_27a) (c_2Emin_2E_3D A_27a)) (c_2Ecombin_2EI A_27a)) (c_2Ecombin_2EI A_27a))) \tag{106}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\
& nonempty A_27c \Rightarrow \forall A_27d.nonempty A_27d \Rightarrow (\forall V0R1 \in (\\
& (2^{A_27a} A_27a). (\forall V1abs1 \in (A_27c^{A_27a}). (\forall V2rep1 \in \\
& (A_27a^{A_27c}). ((p (ap (ap (ap (c_2Equotient_2EQUOTIENT A_27a A_27c) \\
& V0R1) V1abs1) V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b} A_27b). (\forall V4abs2 \in \\
& (A_27d^{A_27b}). (\forall V5rep2 \in (A_27b^{A_27d}). ((p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& A_27b A_27d) V3R2) V4abs2) V5rep2)) \Rightarrow (p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\
& (A_27b^{A_27a}) (A_27d^{A_27c})) (ap (ap (c_2Equotient_2E_3D_3D_3D_3E \\
& A_27a A_27b) V0R1) V3R2)) (ap (ap (c_2Equotient_2E_2D_2D_3E A_27c \\
& A_27b A_27a A_27d) V2rep1) V4abs2)) (ap (ap (c_2Equotient_2E_2D_2D_3E \\
& A_27a A_27d A_27c A_27b) V1abs1) V5rep2)))))))))))))
\end{aligned} \tag{107}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x \in A_27b.(\forall V4y \in \\
& A_27b.((V3x = V4y) \Leftrightarrow (p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3x))\ (ap\ V2rep\ V4y)))))))))) \\
& \hspace{15em} (108)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& A_27a.(\forall V5y1 \in A_27a.(\forall V6y2 \in A_27a.(((p\ (ap\ (ap\ V0R \\
& V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2)))))))))) \\
& \hspace{15em} (109)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27d^{A_27c}). \\
& ((\lambda V7x \in A_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x)))))))))) \\
& \hspace{15em} (110)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0REL \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& A_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2)))))))))) \\
& \hspace{15em} (111)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27b}).((p\ (\\
& ap\ (c_2Ebool_2E_21\ A_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b\ 2)\ V1abs)\ (c_2Ecombin_2EI \\
& \quad 2))\ V3f))))))))) \\
& \hspace{15em} (112)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27a}).(\forall V4g \in \\
& \quad (2^{A_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad 2)\ V0R)\ (c_2Emin_2E_3D\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (\\
& ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0R))\ V4g))))))))) \\
& \hspace{15em} (113)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b^{A_27a}). \\
& \quad (\forall V7g \in (A_27b^{A_27a}).(\forall V8x \in A_27a.(\forall V9y \in \\
& \quad A_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))) \\
& \hspace{15em} (114)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0E \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1P \in (2^{A_27a}).((p\ (ap\ (c_2Equotient_2EEQUIV\ A_27a) \\
& \quad V0E)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E_21\ A_27a)\ V1P)))))) \\
& \hspace{15em} (115)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \hspace{15em} (116)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (117)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (118)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (119)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (120)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((\neg(p V1q)) \vee (\neg(p V2r))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (121)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (122)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (123)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (124)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (125)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (126)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (127)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (128)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (129)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (130)$$

Theorem 1

$$(\forall V0q \in ty_2Einteger_2Eint. ((\neg(V0q = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))) \Rightarrow (\forall V1p \in ty_2Einteger_2Eint. ((V1p = (ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ (ap\ c_2Einteger_2Eint_mul\ (ap\ (ap\ c_2Einteger_2Eint_quot\ V1p)\ V0q))\ V0q))\ (ap\ (ap\ c_2Einteger_2Eint_rem\ V1p)\ V0q))) \wedge ((p\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ 2)\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))\ V1p))\ (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))\ (ap\ (ap\ c_2Einteger_2Eint_rem\ V1p)\ V0q)))\ (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ (ap\ c_2Einteger_2Eint_rem\ V1p)\ V0q))\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)))))) \wedge (p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2EABS\ (ap\ (ap\ c_2Einteger_2Eint_rem\ V1p)\ V0q)))\ (ap\ c_2Einteger_2EABS\ V0q))))))))))$$