

# thm\_2Einteger\_2EINT\_\_REM\_\_ID (TM- bXMQMeEwUFsKsryP26mgixY54sRyF2zwP)

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**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E2ET$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 3** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 4** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 5** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E3D (2^{A-27a}))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ ($

Let  $c\_2Earithmetic\_2E2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 7** We define  $c\_2Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmic\_2EBIT1))$

**Definition 8** We define  $c\_2Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \quad (7)$$

Let  $c\_2Einteger\_2Eint\_rem : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_rem \in ((ty\_2Einteger\_2Eint^{ty\_2Einteger\_2Eint})^{ty\_2Einteger\_2Eint}) \quad (8)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (9)$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint}) \quad (10)$$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the\ (\lambda x.x \in A \wedge p\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E\_40\ ty\_2Einteger\_2Eint))$

Let  $c\_2Einteger\_2Eint\_mul : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum})^{ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Einteger\_2Eint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})} \quad (13)$$

**Definition 11** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$

**Definition 12** We define  $c\_2Einteger\_2Eint\_mul$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint$

Let  $c\_2Einteger\_2Eint\_add : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum})^{ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum}) \quad (14)$$

**Definition 13** We define  $c\_Einteger\_Eint\_add$  to be  $\lambda V0T1 \in ty\_Einteger\_Eint.\lambda V1T2 \in ty\_Einteger$

**Definition 14** We define  $c\_Ebool\_E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_Emin\_E\_40$

Let  $c\_Einteger\_Eint\_lt : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_lt \in ((2^{(ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum\_Eenum)})^{(ty\_Epair\_Eprod\ ty\_Eenum\_Eenum)}) \quad (15)$$

**Definition 15** We define  $c\_Einteger\_Eint\_lt$  to be  $\lambda V0T1 \in ty\_Einteger\_Eint.\lambda V1T2 \in ty\_Einteger$

**Definition 16** We define  $c\_Ebool\_E\_2F$  to be  $(ap\ (c\_Ebool\_E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 17** We define  $c\_Emin\_E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 18** We define  $c\_Ebool\_E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_Emin\_E\_3D\_3D\_3E\ V0t)\ c\_Ebool\_E\_2F$

**Definition 19** We define  $c\_Einteger\_Eint\_le$  to be  $\lambda V0x \in ty\_Einteger\_Eint.\lambda V1y \in ty\_Einteger\_Eint$

Let  $c\_Einteger\_Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_of\_num \in (ty\_Einteger\_Eint^{ty\_Eenum\_Eenum}) \quad (16)$$

**Definition 20** We define  $c\_Ebool\_E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_Ebool\_E\_21\ 2)\ (\lambda V2t \in 2.V2t$

**Definition 21** We define  $c\_Ebool\_ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let  $c\_Einteger\_Eint\_neg : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_neg \in ((ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum\_Eenum)^{(ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum\_Eenum)}) \quad (17)$$

**Definition 22** We define  $c\_Einteger\_Eint\_neg$  to be  $\lambda V0T1 \in ty\_Einteger\_Eint.(ap\ c\_Einteger\_Eint$

**Definition 23** We define  $c\_Einteger\_EABS$  to be  $\lambda V0n \in ty\_Einteger\_Eint.(ap\ (ap\ (ap\ (c\_Ebool\_E\_2E$

**Definition 24** We define  $c\_Ebool\_E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_Ebool\_E\_21\ 2)\ (\lambda V2t \in 2.V2t$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (27)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (28)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (29)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0b \in 2.(\forall V1t \in A.27a. ((ap (ap (ap (c.2Ebool.2ECOND A.27a) V0b) V1t) V1t) = V1t))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge (((\neg(p V0A)) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (35)$$

Assume the following.

$$(\forall V0x \in ty.2Einteger.2Eint.((ap (ap c.2Einteger.2Eint\_add V0x) (ap c.2Einteger.2Eint\_of\_num c.2Enum.2E0)) = V0x)) \quad (36)$$

Assume the following.

$$(\forall V0x \in ty.2Einteger.2Eint.((ap (ap c.2Einteger.2Eint\_mul (ap c.2Einteger.2Eint\_of\_num (ap c.2Earithmic.2ENUMERAL (ap c.2Earithmic.2EBIT1 c.2Earithmic.2EZERO)))) V0x) = V0x)) \quad (37)$$

Assume the following.

$$(\forall V0x \in ty.2Einteger.2Eint.(p (ap (ap c.2Einteger.2Eint\_le V0x) V0x))) \quad (38)$$

Assume the following.

$$(\forall V0x \in ty.2Einteger.2Eint.(\forall V1y \in ty.2Einteger.2Eint. ((p (ap (ap c.2Einteger.2Eint\_le V0x) V1y)) \Leftrightarrow ((p (ap (ap c.2Einteger.2Eint\_lt V0x) V1y)) \vee (V0x = V1y)))))) \quad (39)$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(p (ap (ap c\_2Einteger\_2Eint\_le (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) (ap c\_2Einteger\_2EABS V0p)))) \quad (40)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap c\_2Einteger\_2EABS (ap c\_2Einteger\_2Eint\_of\_num V0n)) = (ap c\_2Einteger\_2Eint\_of\_num V0n))) \quad (41)$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(((ap c\_2Einteger\_2EABS V0p) = (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) \Leftrightarrow (V0p = (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)))) \quad (42)$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1q \in ty\_2Einteger\_2Eint.(\forall V2r \in ty\_2Einteger\_2Eint.(((p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2EABS V2r)) (ap c\_2Einteger\_2EABS V1q))) \wedge ((p (ap (ap (ap (c\_2Ebool\_2ECOND 2) (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0) V0p)) (ap (ap c\_2Einteger\_2Eint\_le (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0) V2r)) (ap (ap c\_2Einteger\_2Eint\_le V2r) (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)))))) \wedge (\exists V3k \in ty\_2Einteger\_2Eint.(V0p = (ap (ap c\_2Einteger\_2Eint\_add (ap (ap c\_2Einteger\_2Eint\_mul V3k) V1q) V2r)))))) \Rightarrow ((ap (ap c\_2Einteger\_2Eint\_rem V0p) V1q) = V2r)))))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (48)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{53}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{54}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{55}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{58}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0p \in ty\_2Einteger\_2Eint. ((\neg(V0p = (ap c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0))) \Rightarrow ((ap (ap c\_2Einteger\_2Eint\_rem V0p) V0p) = (ap \\
& c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0))))
\end{aligned}$$