thm_2Einteger_2EINT__SUB__REDUCE (TMLcA8YMYWBS8cqwQwg5CNJVae1LSjTVZSw)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\textsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\text{-}27a}).(ap\ (ap\ (\textsf{c\_2Emin\_2E\_3D}\ (2^{A\text{-}27}$

Definition 4 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\textsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\textsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{3}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{4}$$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

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Definition 9 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 $(ty\_$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (5)$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum}$$
$$(6)$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}}$$
$$(7)$$

Definition 10 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 11 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty}$$
$$(8)$$

Definition 12 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einte$

Definition 13 *We define* c_2Einteger_2Eint__sub *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (9)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (10)$$

Definition 14 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (11)$$

Assume the following.
$$True \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in$$
$$A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \land (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \land True) \Leftrightarrow$$
$$(p\ V0t)) \land (((False \land (p\ V0t)) \Leftrightarrow False) \land ((((p\ V0t) \land False) \Leftrightarrow False) \land \quad (14)$$
$$(((p\ V0t) \land (p\ V0t)) \Leftrightarrow (p\ V0t)))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \quad (15)$$
$$True))$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_add}$$
$$(ap\ \textsf{c\_2Einteger\_2Eint\_\_of\_\_num}\ \textsf{c\_2Enum\_2E0}))\ V0x) = V0x)) \quad (16)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_add}$$
$$V0x)\ (ap\ \textsf{c\_2Einteger\_2Eint\_\_of\_\_num}\ \textsf{c\_2Enum\_2E0})) = V0x)) \quad (17)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ \textsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ \textsf{c\_2Einteger\_2Eint\_\_neg}\ V0x)) = V0x)) \quad (18)$$

Assume the following.

$$((ap\ \textsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\textsf{c\_2Enum\_2E0})) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \textsf{c\_2Enum\_2E0})) \quad (19)$$

Theorem 1

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Einteger\_2Eint.(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_sub}$$
$$V2p)\ (ap\ c\_2Einteger\_2Eint\_of\_num\ \mathsf{c\_2Enum\_2E0})) = V2p) \wedge (((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_sub}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$\mathsf{c\_2Enum\_2E0})\ V2p) = (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ V2p)) \wedge (((ap\ ($$
$$ap\ \mathsf{c\_2Einteger\_2Eint\_sub}\ (ap\ c\_2Einteger\_2Eint\_of\_num\ (ap$$
$$\mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0m)))\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n))) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_add}$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0m)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n))))) \wedge (((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_sub}$$
$$(ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_of\_num\ ($$
$$ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0m))))\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n))) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_add}$$
$$(ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_of\_num\ ($$
$$ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0m))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1n))))) \wedge (((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_sub}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0m)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1n)))) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_add}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0m)))\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)))) \wedge ((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_sub}$$
$$(ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_of\_num\ ($$
$$ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0m))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1n)))) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0m))))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1n))))))))))))$$