

thm_2Einteger_2ETINT_EQ_EQUIV
 (TMXZGm1U1KQ3mHymCC2Lqe7asHfZ8FADgEN)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A. \lambda P \in (2^{A_{-27}a}). (ap (ap (c_2Emin_2E_3D (2^{A_{-27}a})) (\lambda V0P \in (2^{A_{-27}a}) . P)))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow & nonempty\ (ty_2Epair_2Eprod \\ & A0\ A1) \end{aligned} \quad (2)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (3)$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (p \Rightarrow p Q))))))$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2))))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (6)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (8)$$

Assume the following.

$$(\forall V0x \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0x)\ V0x))) \quad (9)$$

Assume the following.

$$(\forall V0x \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). (\forall V1y \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). ((p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0x)\ V1y)) \Leftrightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V1y)\ V0x)))))) \quad (10)$$

Assume the following.

$$(\forall V0x \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). (\forall V1y \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). (\forall V2z \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). (((p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V1y)\ V2z))) \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0x)\ V2z))))))) \quad (11)$$

Theorem 1

$$(\forall V0p \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). (\forall V1q \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). ((p\ (ap\ (ap\ c_2Einteger_2Etint_eq\ V0p)\ V1q)) \Leftrightarrow ((ap\ c_2Einteger_2Etint_eq\ V0p) = (ap\ c_2Einteger_2Etint_eq\ V1q))))))$$