

thm_2Einteger_2ETINT__MUL__SYM (TM- cHDZXD TBQqWRMwdtJbLTAgKzDiubbAR18)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Eenum_2E_enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2E_enum \tag{1}$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Eenum_2E_enum^{ty_2Eenum_2E_enum})^{ty_2Eenum_2E_enum}) \tag{2}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Eenum_2E_enum^{ty_2Eenum_2E_enum})^{ty_2Eenum_2E_enum}) \tag{3}$$

Let $ty_2Epair_2E_eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2E_eprod\ A0\ A1) \tag{4}$$

Let $c_2Einteger_2E_etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2E_etint_mul \in (((ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum)^{(ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum)})^{(ty_2Epair_2E_eprod\ ty_2Eenum_2E_enum\ ty_2Eenum_2E_enum)}) \tag{5}$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (6)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (7)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b))^{(2^{A_27b})^{A_27a}} \end{aligned} \quad (8)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2E$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ V1n)\ V0m)))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2A \\ V1n)\ V0m)))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ \forall V2p \in ty_2Enum_2Enum. (((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m) \\ V2p) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V2p)) \Leftrightarrow (V0m = V1n)))))) \end{aligned} \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x1 \in ty_2Enum_2Enum. (\forall V1y1 \in ty_2Enum_2Enum. \\
& (\forall V2x2 \in ty_2Enum_2Enum. (\forall V3y2 \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Einteger_2Etint_mul (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V0x1) V1y1)) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V2x2) V3y2)) = (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A \\
& V0x1) V2x2)) (ap (ap c_2Earithmetic_2E_2A V1y1) V3y2))) (ap (ap \\
& c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0x1) V3y2)) \\
& (ap (ap c_2Earithmetic_2E_2A V1y1) V2x2))))))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& A_27b. (((ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y) = (ap (ap \\
& (c_2Epair_2E_2C A_27a A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0x \in (ty_2Epair_2Eprod A_27a A_27b). ((ap (ap (c_2Epair_2E_2C \\
& A_27a A_27b) (ap (c_2Epair_2EFSST A_27a A_27b) V0x)) (ap (c_2Epair_2ESND \\
& A_27a A_27b) V0x)) = V0x))
\end{aligned} \tag{17}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& (\forall V1y \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\
& ((ap (ap c_2Einteger_2Etint_mul V0x) V1y) = (ap (ap c_2Einteger_2Etint_mul \\
& V1y) V0x))))
\end{aligned}$$