

thm_2Einteger_2ETINT__MUL__WELLDEFR
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 sLw3q49jVQk7TN)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty \ ty_2Enum_2Enum \quad (1)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (2)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty \ A0 \Rightarrow \forall A1.nonempty \ A1 \Rightarrow nonempty \ (ty_2Epair_2Eprod \\ A0 \ A1) \end{aligned} \quad (3)$$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod \ ty_2Enum_2Enum \ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod \ ty_2Enum_2Enum \ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod \ ty_2Enum_2Enum \ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod \ ty_2Enum_2Enum \ ty_2Enum_2Enum)} \quad (4)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (5)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)} \quad (6)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a &\Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Epair_2ESND \\ A_27a \ A_27b &\in (A_27b^{(ty_2Epair_2Eprod A_27a \ A_27b)}) \end{aligned} \quad (7)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a &\Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Epair_2EFST \\ A_27a \ A_27b &\in (A_27a^{(ty_2Epair_2Eprod A_27a \ A_27b)}) \end{aligned} \quad (8)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a &\Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a \ A_27b &\in ((ty_2Epair_2Eprod A_27a \ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (9)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2E_2C A_27a A_27b) (V0x V1y))$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (& \\ (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B & \\ V1n) V0m)))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (& \\ \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) & \\ (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B & \\ (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (& \\ \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (ap & \\ (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p) = (ap (ap c_2Earithmetic_2E_2B & \\ (ap (ap c_2Earithmetic_2E_2A V0m) V2p)) (ap (ap c_2Earithmetic_2E_2A & \\ V1n) V2p)))))) \end{aligned} \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. (((\text{True} \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow \text{True}) \Leftrightarrow (p V0t)) \wedge (((\text{False} \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(p V0t))))))) \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0x1 \in ty_2Enum_2Enum. (\forall V1y1 \in ty_2Enum_2Enum. \\ & \quad (\forall V2x2 \in ty_2Enum_2Enum. (\forall V3y2 \in ty_2Enum_2Enum. \\ & \quad ((ap (ap c_2Einteger_2Etint_mul (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\ & \quad ty_2Enum_2Enum) V0x1) V1y1)) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\ & \quad ty_2Enum_2Enum) V2x2) V3y2)) = (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\ & \quad ty_2Enum_2Enum) (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A \\ & \quad V0x1) V2x2)) (ap (ap c_2Earithmetic_2E_2A V1y1) V3y2)) (ap (ap \\ & \quad c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0x1) V3y2)) \\ & \quad (ap (ap c_2Earithmetic_2E_2A V1y1) V2x2)))))))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0x1 \in ty_2Enum_2Enum. (\forall V1y1 \in ty_2Enum_2Enum. \\ & \quad (\forall V2x2 \in ty_2Enum_2Enum. (\forall V3y2 \in ty_2Enum_2Enum. \\ & \quad ((p (ap (ap c_2Einteger_2Etint_eq (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\ & \quad ty_2Enum_2Enum) V0x1) V1y1)) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\ & \quad ty_2Enum_2Enum) V2x2) V3y2)) \Leftrightarrow (ap (ap c_2Earithmetic_2E_2B \\ & \quad V0x1) V3y2)) = (ap (ap c_2Earithmetic_2E_2B V2x2) V1y1))))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \\ & \quad \forall V0x \in (ty_2Epair_2Eprod A_27a A_27b). ((ap (ap (c_2Epair_2E_2C \\ & \quad A_27a A_27b) (ap (c_2Epair_2EFST A_27a A_27b) V0x)) (ap (c_2Epair_2ESND \\ & \quad A_27a A_27b) V0x)) = V0x)) \end{aligned} \quad (18)$$

Theorem 1

$$\begin{aligned} & (\forall V0x1 \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\ & \quad (\forall V1x2 \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\ & \quad (\forall V2y \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\ & \quad ((p (ap (ap c_2Einteger_2Etint_eq V0x1) V1x2)) \Rightarrow (p (ap (ap c_2Einteger_2Etint_eq \\ & \quad (ap (ap c_2Einteger_2Etint_mul V0x1) V2y)) (ap (ap c_2Einteger_2Etint_mul \\ & \quad V1x2) V2y))))))) \end{aligned}$$