# thm_2Einteger_2Eint__ABS__REP__CLASS (TMGvbYthn8qt4LrL2zkAGwK1xcMwMxtvz31)

October 26, 2020

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (2)$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \quad (3)$$

**Definition 1** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 3** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40 $A$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \quad (4)$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \quad (5)$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \quad (6)$$

**Definition 4** *We define* c_2Ebool_2ET *to be* $(ap\ (ap$ (c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

**Definition 5** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

**Definition 6** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 7** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$((\forall V0a \in ty\_2Einteger\_2Eint.((ap\ c\_2Einteger\_2Eint\_\_ABS\_\_CLASS\ (ap\ c\_2Einteger\_2Eint\_\_REP\_\_CLASS\ V0a)) = V0a)) \wedge (\forall V1r \in (2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}).((p\ (ap\ (\lambda V2c \in (2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}).(ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (\lambda V3r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ c\_2Einteger\_2Etint\_\_eq\ V3r)\ V3r))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}))\ V2c)\ (ap\ c\_2Einteger\_2Etint\_\_eq\ V3r))))))\ V1r)) \Leftrightarrow ((ap\ c\_2Einteger\_2Eint\_\_REP\_\_CLASS\ (ap\ c\_2Einteger\_2Eint\_\_ABS\_\_CLASS\ V1r)) = V1r)))) \tag{7}$$

**Theorem 1**

$$((\forall V0a \in ty\_2Einteger\_2Eint.((ap\ c\_2Einteger\_2Eint\_\_ABS\_\_CLASS\ (ap\ c\_2Einteger\_2Eint\_\_REP\_\_CLASS\ V0a)) = V0a)) \wedge (\forall V1c \in (2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}).((\exists V2r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).((p\ (ap\ (ap\ c\_2Einteger\_2Etint\_\_eq\ V2r)\ V2r)) \wedge (V1c = (ap\ c\_2Einteger\_2Etint\_\_eq\ V2r)))) \Leftrightarrow ((ap\ c\_2Einteger\_2Eint\_\_REP\_\_CLASS\ (ap\ c\_2Einteger\_2Eint\_\_ABS\_\_CLASS\ V1c)) = V1c))))$$