thm_2Einteger__word_2EINT__MIN (TMXM4nCXYYXrmxzpDpSG4phSitYehwFEyHyQ)

October 26, 2020

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (3)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (4)$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 13 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{5}$$

Definition 14 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{6}$$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{8}$$

Let $c\_2Einteger\_\_word\_2EINT\_\_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Einteger\_\_word\_2EINT\_\_MAX\ A\_27a \in (ty\_2Einteger\_2Eint^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{9}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{10}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{11}$$

Definition 15 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 (t

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{12}$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{13}$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \tag{14}$$

Definition 16 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 17 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{15}$$

Definition 18 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Intege$

Definition 19 *We define* c_2Einteger_2Eint__sub *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2$

Let $c\_2Einteger\_\_word\_2EINT\_\_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Einteger\_\_word\_2EINT\_\_MIN\ A\_27a \in (ty\_2Einteger\_2Eint^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{16}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{17}$$

Definition 20 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{18}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{19}$$

Definition 21 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Definition 22 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 23 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{20}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \tag{21}$$

Let $c\_2Ewords\_2EINT\_\_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_\_MIN\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27} \tag{22}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ V1n) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))))) \tag{23}$$

Assume the following.

$$(\forall V0b1 \in ty\_2Enum\_2Enum.(\forall V1b2 \in ty\_2Enum\_2Enum. (\forall V2x \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V0b1)\ V2x) = (ap\ (ap\ c\_2Earithmetic\_2EEXP\ V1b2)\ V2x)) \Leftrightarrow ((V2x = \mathsf{c\_2Enum\_2E0}) \lor (V0b1 = V1b2)))))) \tag{24}$$

Assume the following.

$$True \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \lor (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \lor True) \Leftrightarrow True) \land (((False \lor (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \lor False) \Leftrightarrow (p\ V0t)) \land (((p\ V0t) \lor (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{27}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint. ((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_sub}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_sub}\ V1y)\ V0x)))) \tag{28}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)) \Leftrightarrow (V0m = V1n)))) \tag{29}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint. ((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_sub}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_sub}\ V0x)\ V1y))\ V0x) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1y)))) \tag{30}$$

Assume the following.

$$((\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n))\Leftrightarrow(V0m = V1n))))\wedge((\forall V2x \in ty\_2Einteger\_2Eint.(\forall V3y \in ty\_2Einteger\_2Eint.(((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V2x) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V3y))\Leftrightarrow(V2x = V3y))))\wedge(\forall V4n \in ty\_2Enum\_2Enum.(\forall V5m \in ty\_2Enum\_2Enum.((((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V4n) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V5m)))\Leftrightarrow((V4n = \mathsf{c\_2Enum\_2E0})\wedge(V5m = \mathsf{c\_2Enum\_2E0})))\wedge(((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V4n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V5m))\Leftrightarrow((V4n = \mathsf{c\_2Enum\_2E0})\wedge(V5m = \mathsf{c\_2Enum\_2E0}))))))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((ap\ (c\_2Einteger\_\_word\_2EINT\_\_MAX\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_sub}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (c\_2Ewords\_2EINT\_\_MIN\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((ap\ (c\_2Einteger\_\_word\_2EINT\_\_MIN\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_sub}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (c\_2Einteger\_\_word\_2EINT\_\_MAX\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((ap\ (c\_2Ewords\_2EINT\_\_MIN\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)) = (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \tag{34}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((ap\ (c\_2Einteger\_\_word\_2EINT\_\_MIN\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (c\_2Ewords\_2EINT\_\_MIN\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))))$$