

thm_2Einteger__word_2EINT__ZERO__LT__UINT__MAX (TMPFB5ztZrBwaeSYtAeuTjJcEmvBKN4bXuk)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_5C_2E_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{3}$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum\ ty_2Eenum_2Eenum)})ty_2Einteger_2Eint) \tag{4}$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 9 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint$.(ap $(c_2Emin_2E_40$ $(ty_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum ty_2Eenum_2Eenum ty_2Eenum_2Eenum ty_2Eenum_2Eenum)) \quad (5)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum ty_2Eenum_2Eenum)})^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)}) \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{2^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum ty_2Eenum_2Eenum)}} \quad (7)$$

Definition 10 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum ty_2Eenum_2Eenum)$

Definition 11 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint$.(ap $c_2Einteger_2Eint_neg$

Let $c_2Einteger_2Eint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum ty_2Eenum_2Eenum)})^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)}) \quad (8)$$

Definition 12 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint$. $\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint)^{ty_2Eenum_2Eenum} \quad (9)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (10)$$

Let $c_2Einteger_word_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Einteger_word_2EUINT_MAX A_27a \in (ty_2Einteger_2Eint)^{(ty_2Ebool_2Eitself A_27a)} \quad (11)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (12)$$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (13)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (15)$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 15 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 17 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(\\ & ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_of_num \\ & V0n))\ (ap\ c_2Einteger_2Eint_of_num\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ & V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_neg \\ & (ap\ c_2Einteger_2Eint_of_num\ V0n)))\ (ap\ c_2Einteger_2Eint_neg \\ & (ap\ c_2Einteger_2Eint_of_num\ V1m)))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ & V1m)\ V0n))) \wedge (((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_neg \\ & (ap\ c_2Einteger_2Eint_of_num\ V0n)))\ (ap\ c_2Einteger_2Eint_of_num \\ & V1m))) \Leftrightarrow ((\neg(V0n = c_2Enum_2E0)) \vee (\neg(V1m = c_2Enum_2E0)))) \wedge ((p \\ & (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_of_num \\ & V0n))\ (ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num \\ & V1m)))) \Leftrightarrow False)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Einteger_word_2EUINT_MAX\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)) = (ap\ c_2Einteger_2Eint_of_num\ (ap\ (c_2Ewords_2EUINT_MAX\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0)\ (ap\ (c_2Ewords_2EUINT_MAX\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)))) \quad (22)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))\ (ap\ (c_2Einteger_word_2EUINT_MAX\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a))))$$