

# thm\_2Einteger\_\_word\_2EWORD\_\_GEi (TMYrUx- hEBdLvXP7M5KbqtwBgm7WZkezuhsz)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{3}$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint}) \tag{4}$$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^{A.^{27a}}).(ap (ap (c\_2Emin\_2E\_3D (2^{A.^{27a}}))$

**Definition 5** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E\_40 (ty\_2Einteger\_2Eint$

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \tag{5}$$

**Definition 6** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint$

**Definition 7** We define  $c\_Ebool\_2EF$  to be  $(ap (c\_Ebool\_2E\_21\ 2) (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define  $c\_Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_2E\_3D\_3D\_3E V0t) c\_Ebool\_2EF))$

**Definition 10** We define  $c\_Einteger\_2Eint\_le$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2Eint$

**Definition 11** We define  $c\_Einteger\_2Eint\_ge$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2Eint$

Let  $ty\_2Efcf\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcf\_2Efinite\_image\ A0) \quad (6)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (7)$$

Let  $c\_2Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (8)$$

Let  $c\_2Efcf\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcf\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (9)$$

**Definition 12** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21\ 2) (\lambda V2t \in 2.V2t))))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (11)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (12)$$

**Definition 13** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

**Definition 14** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_Emin\_2E\_40\ P))))$

**Definition 15** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 16** We define  $c\_Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ c\_Ebool\_2E\_2F\_5C\ P)))$

**Definition 17** We define  $c\_2Efcf\_2Efinite\_index$  to be  $\lambda A\_27a : \iota.(ap\ (c\_Emin\_2E\_40\ (A\_27a^{ty\_2Enum\_2Enum})))$

Let  $ty\_2Efc\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efc\_2Ecart\ A0\ A1) \quad (13)$$

Let  $c\_2Efc\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efc\_2Edest\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efc\_2Efinite\_image\ A\_27b)})^{(ty\_2Efc\_2Ecart\ A\_27a\ A\_27b)}) \quad (14)$$

**Definition 18** We define  $c\_2Efc\_2Efc\_index$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efc\_2Ecart\ A\_27a\ A\_27b).$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (15)$$

**Definition 19** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP).$

**Definition 20** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0.$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (16)$$

**Definition 21** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n))$

**Definition 22** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (17)$$

**Definition 23** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap\ (ap\ c\_2Ebool\_2ECOND\ t1\ t2)))))$

**Definition 24** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2Ebool\_2ESBIT\ b)\ n)))$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (18)$$

**Definition 25** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c\_2Ewords\_2Ew2n\ w))$

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (19)$$

Let  $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (20)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (21)$$

**Definition 26** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_Enum\_Enum.(ap (ap c\_Earithmetic\_EBIT1))$ .  
Let  $c\_Earithmetic\_EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EDIV \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum})^{ty\_Enum\_Enum} \quad (22)$$

**Definition 27** We define  $c\_Ebit\_EDIV\_EXP$  to be  $\lambda V0x \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum$ .  
Let  $c\_Earithmetic\_EMOD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EMOD \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum})^{ty\_Enum\_Enum} \quad (23)$$

**Definition 28** We define  $c\_Ebit\_EMOD\_EXP$  to be  $\lambda V0x \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum$ .

**Definition 29** We define  $c\_Ebit\_EBITS$  to be  $\lambda V0h \in ty\_Enum\_Enum.\lambda V1l \in ty\_Enum\_Enum.\lambda V2m \in ty\_Enum\_Enum$ .

**Definition 30** We define  $c\_Ebit\_EBIT$  to be  $\lambda V0b \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum.(ap (ap c\_Ebit\_EBIT))$ .

**Definition 31** We define  $c\_Efc\_EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_Enum\_Enum}).(ap (ap c\_Efc\_EFCP)))$ .

**Definition 32** We define  $c\_Ewords\_En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_Enum\_Enum.(ap (c\_Efc\_EFCP))$ .

**Definition 33** We define  $c\_Ewords\_Eword\_comp$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_Efc\_Ecart\ 2\ A\_27a)$ .

Let  $c\_Einteger\_Etint\_neg : \iota$  be given. Assume the following.

$$c\_Einteger\_Etint\_neg \in ((ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum)^{ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum})^{ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum} \quad (24)$$

Let  $c\_Einteger\_Etint\_eq : \iota$  be given. Assume the following.

$$c\_Einteger\_Etint\_eq \in ((2^{(ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum)})^{ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum})^{ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum} \quad (25)$$

Let  $c\_Einteger\_Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_Einteger\_Eint\_ABS\_CLASS \in (ty\_Einteger\_Eint)^{(2^{(ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum)})^{ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum}} \quad (26)$$

**Definition 34** We define  $c\_Einteger\_Eint\_ABS$  to be  $\lambda V0r \in (ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum)$ .

**Definition 35** We define  $c\_Einteger\_Eint\_neg$  to be  $\lambda V0T1 \in ty\_Einteger\_Eint.(ap c\_Einteger\_Eint)$ .

**Definition 36** We define  $c\_Ewords\_Eword\_msb$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_Efc\_Ecart\ 2\ A\_27a).(ap (ap c\_Ewords\_Eword\_msb))$ .

**Definition 37** We define  $c\_Einteger\_word\_Ew2i$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_Efc\_Ecart\ 2\ A\_27a).(ap (ap c\_Einteger\_word\_Ew2i))$ .

**Definition 38** We define  $c\_Ebool\_E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_E21\ 2)) (\lambda V2t \in 2)))$ .

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b} A\_27a)}) \end{aligned} \quad (27)$$

**Definition 39** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

**Definition 40** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

**Definition 41** We define  $c\_2Ewords\_2Eenzcv$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1b \in ($

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (28)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (29)$$

**Definition 42** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27$

**Definition 43** We define  $c\_2Ewords\_2Eword\_le$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1b$

**Definition 44** We define  $c\_2Ewords\_2Eword\_ge$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1b$

Assume the following.

$$True \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Efc\_2Ecart \\ 2\ A\_27a).(\forall V1b \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).((p\ (ap\ (ap \\ (c\_2Ewords\_2Eword\_le\ A\_27a)\ V0a)\ V1b)) \Leftrightarrow (p\ (ap\ (ap\ c\_2Einteger\_2Eint\_le \\ (ap\ (c\_2Einteger\_word\_2Ew2i\ A\_27a)\ V0a))\ (ap\ (c\_2Einteger\_word\_2Ew2i \\ A\_27a)\ V1b)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0a \in (ty\_2Efc\_2Ecart \\ 2\ A_{.27a}).(\forall V1b \in (ty\_2Efc\_2Ecart\ 2\ A_{.27a}).((p\ (ap\ (ap \\ (c\_2Ewords\_2Eword\_ge\ A_{.27a})\ V0a)\ V1b)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ewords\_2Eword\_le \\ A_{.27a})\ V1b)\ V0a)))))) \end{aligned} \quad (34)$$

**Theorem 1**

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0a \in (ty\_2Efc\_2Ecart \\ 2\ A_{.27a}).(\forall V1b \in (ty\_2Efc\_2Ecart\ 2\ A_{.27a}).((p\ (ap\ (ap \\ (c\_2Ewords\_2Eword\_ge\ A_{.27a})\ V0a)\ V1b)) \Leftrightarrow (p\ (ap\ (ap\ c\_2Einteger\_2Eint\_ge \\ (ap\ (c\_2Einteger\_word\_2Ew2i\ A_{.27a})\ V0a))\ (ap\ (c\_2Einteger\_word\_2Ew2i \\ A_{.27a})\ V1b)))))) \end{aligned}$$