

thm_2Einteger__word_2EWORD__LEi
(TMFS2vZYPEAiCU5NvHSKRi9YQYmfutueA4r)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{3}$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^{A.^{27a}}).(ap (ap (c_2Emin_2E_3D (2^{A.^{27a}}))$

Definition 5 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty_2Einteger_2Eint$

Let $c_2Einteger_2Eint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \tag{5}$$

Definition 6 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 7 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2EF))$

Definition 10 We define $c_Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$

Let $ty_2Efcp_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efcp_2Efinite_image\ A0) \quad (6)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (7)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (8)$$

Let $c_2Efcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efcp_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (9)$$

Definition 11 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (12)$$

Definition 12 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Definition 13 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_Emin_2E_40\ P))))$

Definition 14 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_Ebool_2E_2F_5C\ P))))$

Definition 16 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_Emin_2E_40\ (A_27a^{ty_2Enum_2Enum})))$

Let $ty_2Efc_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efc_2Ecart\ A0\ A1) \quad (13)$$

Let $c_2Efc_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efc_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efc_2Efinite_image\ A_27b)})^{(ty_2Efc_2Ecart\ A_27a\ A_27b)}) \quad (14)$$

Definition 17 We define $c_2Efc_2Efc_index$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in (ty_2Efc_2Ecart\ A_27a\ A_27b).$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (15)$$

Definition 18 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP).$

Definition 19 We define $c_2Earithmetic_2EZERO$ to be $c_2Enum_2E0.$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (16)$$

Definition 20 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 21 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x.$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (17)$$

Definition 22 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap\ (ap\ c_2Ebool_2ECOND\ t1\ t2))\ t2))\ t1)$

Definition 23 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2. \lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2ESBIT\ b)\ n)))$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 24 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap\ (ap\ c_2Esum_num_2ESUM\ w))$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (20)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (21)$$

Definition 25 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_Enum_Enum.(ap (ap c_Earithmetic_EBIT1))$.
Let $c_Earithmetic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_EDIV \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (22)$$

Definition 26 We define $c_Ebit_EDIV_EXP$ to be $\lambda V0x \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$.
Let $c_Earithmetic_EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_EMOD \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (23)$$

Definition 27 We define $c_Ebit_EMOD_EXP$ to be $\lambda V0x \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$.

Definition 28 We define c_Ebit_EBITS to be $\lambda V0h \in ty_Enum_Enum.\lambda V1l \in ty_Enum_Enum.\lambda V2m \in ty_Enum_Enum$.

Definition 29 We define c_Ebit_EBIT to be $\lambda V0b \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum.(ap (ap c_Ebit_EBIT))$.

Definition 30 We define c_Efc_EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_Enum_Enum}).(ap (ap c_Efc_EFCP)))$.

Definition 31 We define c_Ewords_En2w to be $\lambda A_27a : \iota.\lambda V0n \in ty_Enum_Enum.(ap (c_Efc_EFCP))$.

Definition 32 We define $c_Ewords_Eword_comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efc_Ecart\ 2\ A_27a)$.

Let $c_Einteger_Etint_neg : \iota$ be given. Assume the following.

$$c_Einteger_Etint_neg \in ((ty_Epair_Eprod\ ty_Enum_Enum\ ty_Enum_Enum)^{(ty_Epair_Eprod\ ty_Enum_Enum\ ty_Enum_Enum)}) \quad (24)$$

Let $c_Einteger_Etint_eq : \iota$ be given. Assume the following.

$$c_Einteger_Etint_eq \in ((2^{(ty_Epair_Eprod\ ty_Enum_Enum\ ty_Enum_Enum)})^{(ty_Epair_Eprod\ ty_Enum_Enum\ ty_Enum_Enum)}) \quad (25)$$

Let $c_Einteger_Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Einteger_Eint_ABS_CLASS \in (ty_Einteger_Eint)^{(2^{(ty_Epair_Eprod\ ty_Enum_Enum\ ty_Enum_Enum)})} \quad (26)$$

Definition 33 We define $c_Einteger_Eint_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Enum_Enum\ ty_Enum_Enum)$.

Definition 34 We define $c_Einteger_Eint_neg$ to be $\lambda V0T1 \in ty_Einteger_Eint.(ap\ c_Einteger_Eint)$.

Definition 35 We define $c_Ewords_Eword_msb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efc_Ecart\ 2\ A_27a).(ap\ c_Ewords_Eword_comp)$.

Definition 36 We define $c_Einteger_word_Ew2i$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efc_Ecart\ 2\ A_27a).(ap\ c_Einteger_Eint)$.

Definition 37 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E_21\ 2)\ (\lambda V2t \in 2))))$.

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (27)$$

Definition 38 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Definition 39 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b$

Definition 40 We define $c_2Ewords_2Eenzcv$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1b \in ($

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (28)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (29)$$

Definition 41 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27$

Definition 42 We define $c_2Ewords_2Eword_lt$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1b$

Definition 43 We define $c_2Ewords_2Eword_le$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1b$

Assume the following.

$$True \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. \\ ((p\ (ap\ (ap\ c_2Einteger_2Eint_le\ V0x)\ V1y)) \Leftrightarrow ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\ V0x)\ V1y)) \vee (V0x = V1y)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0v \in (ty_2EfcP_2Ecart \\ 2\ A_27a).(\forall V1w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(((ap\ (c_2Einteger_word_2Ew2i \\ A_27a)\ V0v) = (ap\ (c_2Einteger_word_2Ew2i\ A_27a)\ V1w)) \Leftrightarrow (V0v = \\ V1w)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2EfcP_2Ecart \\ 2\ A_27a).(\forall V1b \in (ty_2EfcP_2Ecart\ 2\ A_27a).((p\ (ap\ (ap \\ (c_2Ewords_2Eword_lt\ A_27a)\ V0a)\ V1b)) \Leftrightarrow (p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\ (ap\ (c_2Einteger_word_2Ew2i\ A_27a)\ V0a))\ (ap\ (c_2Einteger_word_2Ew2i \\ A_27a)\ V1b))))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2EfcP_2Ecart \\ 2\ A_27a).(\forall V1b \in (ty_2EfcP_2Ecart\ 2\ A_27a).((p\ (ap\ (ap \\ (c_2Ewords_2Eword_le\ A_27a)\ V0a)\ V1b)) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ewords_2Eword_lt \\ A_27a)\ V0a)\ V1b)) \vee (V0a = V1b)))))) \end{aligned} \quad (36)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2EfcP_2Ecart \\ 2\ A_27a).(\forall V1b \in (ty_2EfcP_2Ecart\ 2\ A_27a).((p\ (ap\ (ap \\ (c_2Ewords_2Eword_le\ A_27a)\ V0a)\ V1b)) \Leftrightarrow (p\ (ap\ (ap\ c_2Einteger_2Eint_le \\ (ap\ (c_2Einteger_word_2Ew2i\ A_27a)\ V0a))\ (ap\ (c_2Einteger_word_2Ew2i \\ A_27a)\ V1b))))))) \end{aligned}$$