

thm_2Einteger__word_2EfromString__def (TMRJYayCZFP66GDb85tzgjhG54sjBFNSbVG)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{3}$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{ty_2Einteger_2Eint}) \tag{4}$$

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2ET$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_2E21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

Definition 5 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E_40\ (ty_2Einteger_2Eint_REP_CLASS\ a)))$

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \tag{5}$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}))(ty_2Epair_2Eprod\ ty_2Enum) \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (7)$$

Definition 6 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum)$

Definition 7 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint_ABS)$

Definition 8 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 11 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.)))$

Definition 12 We define $c_2Ebool_2Eliteral_case$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b.))$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (8)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (10)$$

Definition 13 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 14 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (12)$$

Definition 15 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ c_2Enum_2ESUC_REP)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (13)$$

Definition 16 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 17 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 18 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 19 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 20 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A$

Definition 21 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Let $ty_2Estring_2Echar : \iota$ be given. Assume the following.

$$nonempty\ ty_2Estring_2Echar \quad (14)$$

Let $c_2Estring_2ECHR : \iota$ be given. Assume the following.

$$c_2Estring_2ECHR \in (ty_2Estring_2Echar)^{ty_2Enum_2Enum} \quad (15)$$

Definition 22 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Definition 23 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 24 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_21$

Definition 25 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1$

Definition 26 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b$

Definition 27 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 28 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 29 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 30 We define $c_2EASCIINumbers_2EUNHEX$ to be $(ap (ap (c_2Erelation_2EWFREC ty_2Estring$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (16)$$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow c_2Elist_2EREVERSE \ A_27a \in ((ty_2Elist_2Elist \ A_27a)_{(ty_2Elist_2Elist \ A_27a)}) \quad (17)$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow c_2Elist_2EMAP \ A_27a \ A_27b \in (((ty_2Elist_2Elist \ A_27b)_{(ty_2Elist_2Elist \ A_27a)})_{(A_27b^{A_27a}})) \quad (18)$$

Let $c_2Enumposrep_2El2n : \iota$ be given. Assume the following.

$$c_2Enumposrep_2El2n \in ((ty_2Enum_2Enum)_{(ty_2Elist_2Elist \ ty_2Enum_2Enum)})_{ty_2Enum_2Enum} \quad (19)$$

Definition 31 We define $c_2EASCIInumbers_2Es2n$ to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1f \in (ty_2Enum_2Enum)$

Definition 32 We define $c_2EASCIInumbers_2Enum_from_dec_string$ to be $(ap \ (ap \ c_2EASCIInumbers_2Es2n))$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint)_{ty_2Enum_2Enum} \quad (20)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow c_2Elist_2ECONS \ A_27a \in (((ty_2Elist_2Elist \ A_27a)_{(ty_2Elist_2Elist \ A_27a)})_{A_27a}) \quad (21)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow c_2Elist_2ENIL \ A_27a \in (ty_2Elist_2Elist \ A_27a) \quad (22)$$

Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow c_2Elist_2Elist_CASE \ A_27a \ A_27b \in (((A_27b)_{(A_27b)_{(ty_2Elist_2Elist \ A_27a)}^{A_27a}})_{A_27b})_{(ty_2Elist_2Elist \ A_27a)} \quad (23)$$

Definition 33 We define $c_2Einteger_word_2EfromString$ to be $(ap \ (ap \ (c_2Erelation_2EWFREC \ (ty_2Elist_2Elist_2Elist_2Elist))))$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (24)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (25)$$

Definition 34 We define $c_2Eprim_rec_2E.3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 35 We define $c_2Earithmic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 36 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 37 We define $c_2Earithmic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 38 We define $c_2Earithmic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 39 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E$

Let $c_2Earithmic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (26)$$

Let $c_2Earithmic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (27)$$

Let $c_2Earithmic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (28)$$

Definition 40 We define $c_2Eenumerat_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 41 We define $c_2ERelation_2EEMPTY_REL$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27a.c_2E$

Let $c_2Estring_2EORD : \iota$ be given. Assume the following.

$$c_2Estring_2EORD \in (ty_2Enum_2Enum^{ty_2Estring_2Echar}) \quad (29)$$

Assume the following.

$$True \quad (30)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\ & A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\ & V0t1) V1t2) = V1t2)))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}).(\forall V1x \in A_27a.((ap\ (ap\ (c_2Ebool_2Eliteral_case \\ & \quad A_27a\ A_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \end{aligned} \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((ap\ (c_2Ecombin_2El \\ \quad A_27a)\ V0x) = V0x)) \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad (\forall V0v \in A_27b.(\forall V1f \in ((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}). \\ & \quad ((ap\ (ap\ (ap\ (c_2Elist_2Elist_CASE\ A_27a\ A_27b)\ (c_2Elist_2ENIL \\ & \quad \quad A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2a0 \in A_27a.(\forall V3a1 \in \\ & \quad \quad (ty_2Elist_2Elist\ A_27a).(\forall V4v \in A_27b.(\forall V5f \in (\\ & \quad \quad \quad (A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}).((ap\ (ap\ (ap\ (c_2Elist_2Elist_CASE \\ & \quad \quad \quad A_27a\ A_27b)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2a0)\ V3a1))\ V4v)\ V5f) = \\ & \quad \quad \quad (ap\ (ap\ V5f\ V2a0)\ V3a1)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum.(\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c_2Earithmic_2EBIT1\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO)\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO) \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& V0n)\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))))))))) \\
\end{aligned} \tag{39}$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (p\ (ap\ (c_2Erelation_2EWF\ A_27a) \\
(c_2Erelation_2EEMPTY_REL\ A_27a))) \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow (\\
& \forall V0M \in ((A_27b^{A_27a})^{(A_27b^{A_27a})}). (\forall V1R \in ((2^{A_27a})^{A_27a}). \\
& (\forall V2f \in (A_27b^{A_27a}). ((V2f = (ap\ (ap\ (c_2Erelation_2EWFREC \\
& A_27a\ A_27b)\ V1R)\ V0M)) \Rightarrow ((p\ (ap\ (c_2Erelation_2EWF\ A_27a)\ V1R)) \Rightarrow \\
& (\forall V3x \in A_27a. ((ap\ V2f\ V3x) = (ap\ (ap\ V0M\ (ap\ (ap\ (ap\ (c_2Erelation_2ERESTRICT \\
& A_27a\ A_27b)\ V2f)\ V1R)\ V3x))))))))) \\
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0r) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
& (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))))))) \Leftrightarrow \\
& ((ap c_2Estring_2EORD (ap c_2Estring_2ECHR V0r) = V0r)))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0c1 \in ty_2Estring_2Echar. (\forall V1c2 \in ty_2Estring_2Echar. \\
& ((V0c1 = V1c2) \Leftrightarrow ((ap c_2Estring_2EORD V0c1) = (ap c_2Estring_2EORD \\
& V1c2))))
\end{aligned} \tag{43}$$

Theorem 1

$$\begin{aligned}
& (\forall V0t \in (ty_2Elist_2Elist\ ty_2Estring_2Echar).(\forall V1v4 \in \\
& ty_2Estring_2Echar.(\forall V2v1 \in (ty_2Elist_2Elist\ ty_2Estring_2Echar). \\
& (((ap\ c_2Einteger_word_2EfromString\ (ap\ (ap\ (c_2Elist_2ECONS \\
& ty_2Estring_2Echar)\ (ap\ c_2Estring_2ECHR\ (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT2 \\
& c_2Earithmetic_2EZERO))))))))))\ V0t)) = (ap\ c_2Einteger_2Eint_neg \\
& (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2EASCIInumbers_2Enum_from_dec_string \\
& V0t))))\wedge(((ap\ c_2Einteger_word_2EfromString\ (ap\ (ap\ (c_2Elist_2ECONS \\
& ty_2Estring_2Echar)\ (ap\ c_2Estring_2ECHR\ (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))))))) \\
& V0t)) = (ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num \\
& (ap\ c_2EASCIInumbers_2Enum_from_dec_string\ V0t))))\wedge(((\\
& ap\ c_2Einteger_word_2EfromString\ (c_2Elist_2ENIL\ ty_2Estring_2Echar)) = \\
& (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2EASCIInumbers_2Enum_from_dec_string \\
& (c_2Elist_2ENIL\ ty_2Estring_2Echar))))\wedge((ap\ c_2Einteger_word_2EfromString \\
& (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Estring_2Echar)\ V1v4)\ V2v1)) = (ap \\
& (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Einteger_2Eint)\ (ap\ (ap\ (c_2Emin_2E3D \\
& ty_2Estring_2Echar)\ V1v4)\ (ap\ c_2Estring_2ECHR\ (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT2 \\
& c_2Earithmetic_2EZERO))))))))))\ (ap\ c_2Einteger_2Eint_neg \\
& (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2EASCIInumbers_2Enum_from_dec_string \\
& V2v1))))\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Einteger_2Eint)\ (ap \\
& (ap\ (c_2Emin_2E3D\ ty_2Estring_2Echar)\ V1v4)\ (ap\ c_2Estring_2ECHR \\
& (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))))))))))\ (ap\ c_2Einteger_2Eint_neg \\
& (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2EASCIInumbers_2Enum_from_dec_string \\
& V2v1))))\ (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2EASCIInumbers_2Enum_from_dec_string \\
& (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Estring_2Echar)\ V1v4)\ V2v1))))))))))
\end{aligned}$$