

thm_2Einteger__word_2EfromString__ind (TMbtP4HufPNgopdqrSef1XvWTW7RfpyZCya)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define c_2Ebool_2E21 to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V0x \in 2.V0x))$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 6 We define `c_2Earithmic_2EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2EBIT1))$

Definition 7 We define `c_2Earithmic_2EZERO` to be `c_2Enum_2E0`.

Definition 8 We define `c_2Earithmic_2EBIT2` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2EBIT2))$

Definition 9 We define `c_2Earithmic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let `ty_2Estring_2Echar` : ι be given. Assume the following.

$$nonempty\ ty_2Estring_2Echar \quad (7)$$

Let `c_2Estring_2ECHR` : ι be given. Assume the following.

$$c_2Estring_2ECHR \in (ty_2Estring_2Echar^{ty_2Enum_2Enum}) \quad (8)$$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (9)$$

Let `c_2Elist_2ECONS` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ECONS\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})^{A.27a}) \quad (10)$$

Definition 10 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define `c_2Ebool_2E_3F` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A.27a))))$

Let `c_2Elist_2ENIL` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ENIL\ A.27a \in (ty_2Elist_2Elist\ A.27a) \quad (11)$$

Definition 12 We define `c_2Ebool_2E_7E` to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 We define `c_2Erelation_2EEMPTY_REL` to be $\lambda A.27a : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27a.c_2Ebool_2E_21$

Definition 14 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 15 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21))$

Definition 16 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 17 We define `c_2Erelation_2EWF` to be $\lambda A.27a : \iota.\lambda V0R \in ((2^{A.27a})^{A.27a}).(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V1t \in 2.V1t))$

Definition 18 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (16)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (17)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(V0x = V0x)) \quad (18)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ (2^{A_27a}). ((\exists V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow \\ ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A_27a. (p\ (\\ ap\ V1Q\ V4x)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ 2. ((\exists V2x \in A_27a. (p\ (ap\ V0P\ V2x))) \vee (p\ V1Q)) \Leftrightarrow (\exists V3x \in \\ A_27a. ((p\ (ap\ V0P\ V3x)) \vee (p\ V1Q)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). ((p\ V0P) \vee (\exists V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow (\exists V3x \in \\ A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ 2. ((\exists V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in \\ A_27a. (p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ A_27a). ((V0l = (c_2Elist_2ENIL\ A_27a)) \vee (\exists V1h \in A_27a. (\\ \exists V2t \in (ty_2Elist_2Elist\ A_27a). (V0l = (ap\ (ap\ (c_2Elist_2ECONS \\ A_27a)\ V1h)\ V2t)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\ ((p\ (ap\ (c_2Erelation_2EWF\ A_27a)\ V0R)) \Rightarrow (\forall V1P \in (2^{A_27a}). \\ ((\forall V2x \in A_27a. ((\forall V3y \in A_27a. ((p\ (ap\ (ap\ V0R\ V3y)\ V2x)) \Rightarrow \\ (p\ (ap\ V1P\ V3y)))))) \Rightarrow (p\ (ap\ V1P\ V2x)))))) \Rightarrow (\forall V4x \in A_27a. (p\ (ap \\ V1P\ V4x)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (c_2Erelation_2EWF\ A_27a) \\ (c_2Erelation_2EEMPTY_REL\ A_27a))) \end{aligned} \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (41)$$

Theorem 1

$$\begin{aligned} & (\forall V0P \in (2^{(ty_2Elist_2Elist\ ty_2Estring_2Echar)}), ((\\ & \quad (\forall V1t \in (ty_2Elist_2Elist\ ty_2Estring_2Echar). (p (ap V0P \\ & \quad (ap (ap (c_2Elist_2ECONS\ ty_2Estring_2Echar) (ap c_2Estring_2ECHR \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT2 \\ & (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT2 \\ & (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))))))) V1t)))) \wedge \\ & \quad ((\forall V2t \in (ty_2Elist_2Elist\ ty_2Estring_2Echar). (p (ap \\ & V0P (ap (ap (c_2Elist_2ECONS\ ty_2Estring_2Echar) (ap c_2Estring_2ECHR \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT2 \\ & (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))))))))) V2t)))) \wedge ((p (ap V0P (c_2Elist_2ENIL \\ & ty_2Estring_2Echar))) \wedge (\forall V3v4 \in ty_2Estring_2Echar. (\\ & \quad \forall V4v1 \in (ty_2Elist_2Elist\ ty_2Estring_2Echar). (p (ap V0P \\ & (ap (ap (c_2Elist_2ECONS\ ty_2Estring_2Echar) V3v4) V4v1)))))) \Rightarrow \\ & \quad (\forall V5v \in (ty_2Elist_2Elist\ ty_2Estring_2Echar). (p (ap V0P \\ & \quad V5v)))))) \end{aligned}$$