

thm_2Einteger__word_2Ei2w__INT__MIN (TMXMwvGFFStGf4XYjsjVvS8ksK4K6CiEZt3)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{3}$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \tag{4}$$

Definition 4 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})$

Definition 6 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty_2Emin_2E_40$

Let $c_2Einteger_2Eint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum})^{ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum}) \tag{5}$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (7)$$

Definition 7 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 8 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (8)$$

Definition 9 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (9)$$

Definition 10 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (10)$$

Definition 11 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint)^{ty_2Enum_2Enum} \quad (11)$$

Definition 12 We define $c_2Einteger_2ENum$ to be $\lambda V0i \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E40\ ty_2Eint$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (13)$$

Definition 13 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 14 We define $c_2\text{Earithmetic_EZERO}$ to be $c_2\text{Enum_E0}$.

Let $c_2\text{Enum_EREP_num} : \iota$ be given. Assume the following.

$$c_2\text{Enum_EREP_num} \in (\text{omega}^{ty_2\text{Enum_2Enum}}) \quad (14)$$

Let $c_2\text{Enum_ESUC_REP} : \iota$ be given. Assume the following.

$$c_2\text{Enum_ESUC_REP} \in (\text{omega}^{\text{omega}}) \quad (15)$$

Definition 15 We define $c_2\text{Enum_ESUC}$ to be $\lambda V0m \in ty_2\text{Enum_2Enum} . (ap\ c_2\text{Enum_EABS_num})$

Definition 16 We define $c_2\text{Earithmetic_EBIT1}$ to be $\lambda V0n \in ty_2\text{Enum_2Enum} . (ap\ (ap\ c_2\text{Earithmetic_EABS_num}))$

Definition 17 We define $c_2\text{Earithmetic_ENUMERAL}$ to be $\lambda V0x \in ty_2\text{Enum_2Enum} . V0x$.

Definition 18 We define $c_2\text{Earithmetic_EBIT2}$ to be $\lambda V0n \in ty_2\text{Enum_2Enum} . (ap\ (ap\ c_2\text{Earithmetic_EABS_num}))$

Let $c_2\text{Earithmetic_EEXP} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_EEXP} \in ((ty_2\text{Enum_2Enum}^{ty_2\text{Enum_2Enum}})^{ty_2\text{Enum_2Enum}}) \quad (16)$$

Let $c_2\text{Earithmetic_EDIV} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_EDIV} \in ((ty_2\text{Enum_2Enum}^{ty_2\text{Enum_2Enum}})^{ty_2\text{Enum_2Enum}}) \quad (17)$$

Definition 19 We define $c_2\text{Ebit_EDIV_EXP}$ to be $\lambda V0x \in ty_2\text{Enum_2Enum} . \lambda V1n \in ty_2\text{Enum_2Enum} . (ap\ c_2\text{Earithmetic_EDIV})$

Let $c_2\text{Earithmetic_E2D} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_E2D} \in ((ty_2\text{Enum_2Enum}^{ty_2\text{Enum_2Enum}})^{ty_2\text{Enum_2Enum}}) \quad (18)$$

Let $c_2\text{Earithmetic_EMOD} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_EMOD} \in ((ty_2\text{Enum_2Enum}^{ty_2\text{Enum_2Enum}})^{ty_2\text{Enum_2Enum}}) \quad (19)$$

Definition 20 We define $c_2\text{Ebit_EMOD_EXP}$ to be $\lambda V0x \in ty_2\text{Enum_2Enum} . \lambda V1n \in ty_2\text{Enum_2Enum} . (ap\ c_2\text{Earithmetic_EMOD})$

Definition 21 We define $c_2\text{Ebit_EBITS}$ to be $\lambda V0h \in ty_2\text{Enum_2Enum} . \lambda V1l \in ty_2\text{Enum_2Enum} . \lambda V$

Definition 22 We define $c_2\text{Ebit_EBIT}$ to be $\lambda V0b \in ty_2\text{Enum_2Enum} . \lambda V1n \in ty_2\text{Enum_2Enum} . (ap\ c_2\text{Ebit_EBITS})$

Let $ty_2\text{EfcP_Efinite_image} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0 . \text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2\text{EfcP_Efinite_image } A0) \quad (20)$$

Let $ty_2\text{Ebool_Eitself} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0 . \text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2\text{Ebool_Eitself } A0) \quad (21)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (22)$$

Let $c_2Efcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efcp_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (23)$$

Definition 23 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 24 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 25 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E))$

Definition 26 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 27 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 28 We define $c_2Ebool_2E3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_2Ebool_2E2F_5C\ P)))$

Definition 29 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_2Emin_2E40\ (A_27a^{ty_2Enum_2Enum})))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcp_2Ecart\ A0\ A1) \quad (24)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efcp_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image\ A_27b)})^{(ty_2Efcp_2Ecart\ A_27a\ A_27b)}) \quad (25)$$

Definition 30 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart\ A_27a\ A_27b)$

Definition 31 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap\ (c_2Efcp_2Efcp_index\ A_27a\ A_27b\ g)))$

Definition 32 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Efcp_2EFCP\ A_27a\ n))$

Definition 33 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(c_2Ebool_2E7E\ t1\ t2))))$

Definition 34 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ b)\ V1n)))$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (26)$$

Let $c_2Enumeral_2EiSUB : \iota$ be given. Assume the following.

$$c_2Enumeral_2EiSUB \in (((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^2) \quad (33)$$

Assume the following.

$$True \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge \\ & ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (44)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. ((p (ap (ap (ap c_2Einteger_2Eint_lt V0x) V1y)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le (ap (ap c_2Einteger_2Eint_add V0x) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) V1y)))))) \quad (45)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_le V0x) V1y)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0) (ap (ap c_2Einteger_2Eint_add V1y) (ap c_2Einteger_2Eint_neg V0x)))))) \quad (46)$$

Assume the following.

$$(\forall V0y \in ty_2Einteger_2Eint.(\forall V1x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add V1x) V0y) = (ap (ap c_2Einteger_2Eint_add V0y) V1x)))) \quad (47)$$

Assume the following.

$$(\forall V0z \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. (\forall V2x \in ty_2Einteger_2Eint.((ap (ap c_2Einteger_2Eint_add V2x) (ap (ap c_2Einteger_2Eint_add V1y) V0z)) = (ap (ap c_2Einteger_2Eint_add (ap (ap c_2Einteger_2Eint_add V2x) V1y)) V0z)))))) \quad (48)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num c_2Enum_2E0) V0x) = V0x)) \quad (49)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.((ap (ap c_2Einteger_2Eint_add V0x) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0) = V0x)) \quad (50)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.((ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x)) \quad (51)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& (ap (ap c_2Einteger_2Eint_add V0x) V1y)) V2z) = (ap (ap c_2Einteger_2Eint_add \\
& (ap (ap c_2Einteger_2Eint_mul V0x) V2z)) (ap (ap c_2Einteger_2Eint_mul \\
& V1y) V2z))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_add V0x) \\
& V1y)) = (ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& V0x)) (ap c_2Einteger_2Eint_neg V1y))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0x) = (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_mul V0x) \\
& V1y)) = (ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_neg \\
& V0x)) V1y))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_mul V0x) \\
& V1y)) = (ap (ap c_2Einteger_2Eint_mul V0x) (ap c_2Einteger_2Eint_neg \\
& V1y))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((\neg(p (ap (ap c_2Einteger_2Eint_le V0x) V1y))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt \\
& V1y) V0x))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((p (ap (ap c_2Einteger_2Eint_lt V0x) V1y))) \Rightarrow (p (ap (ap c_2Einteger_2Eint_le \\
& V0x) V1y))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Einteger_2Eint. (\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2m \in ty_2Enum_2Enum. (((ap (ap c_2Einteger_2Eint_add \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0p) = V0p) \wedge (((\\
& ap (ap c_2Einteger_2Eint_add V0p) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) = V0p) \wedge (((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \wedge \\
& (((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_neg V0p)) = \\
& V0p) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V1n))) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V2m))) = (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B \\
& V1n) V2m)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V1n))) (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V2m)))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap \\
& (ap c_2Earithmetic_2E_3C_3D V2m) V1n)) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V1n) \\
& V2m)))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V2m) \\
& V1n)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1n))) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V2m))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap (\\
& ap c_2Earithmetic_2E_3C_3D V1n) V2m)) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V2m) \\
& V1n)))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V1n) \\
& V2m)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1n))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V2m)))) = (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B V1n) V2m))))))))))))))
\end{aligned}$$

(59)

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& \quad ((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow \\
& \quad True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V0n)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT1 V0n)))))) \Leftrightarrow \\
& \quad False) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT2 V0n)))))) \Leftrightarrow \\
& \quad False) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT1 V0n)))) \\
& \quad \quad (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow False) \wedge (((p \\
& \quad \quad (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT2 V0n)))) \\
& \quad \quad (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow False) \wedge (((p \\
& \quad \quad (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_neg (ap \\
& \quad \quad c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL (ap \\
& \quad \quad c_2Arithmetic_2EBIT1 V0n)))) (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap \\
& \quad \quad c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad (ap c_2Arithmetic_2EBIT2 V0n)))) (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap \\
& \quad \quad c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL V0n))) \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V1m)))) \Leftrightarrow (p (ap (ap c_2Arithmetic_2E_3C_3D V0n) V1m))) \wedge (((p (\\
& \quad \quad ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL V0n))) (ap c_2Integer_2Eint_neg \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad (ap c_2Arithmetic_2EBIT1 V1m)))))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Integer_2Eint_le \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V0n))) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT2 V1m)))))) \Leftrightarrow \\
& \quad False) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_neg \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V0n)))) (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V1m)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_neg \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V0n)))) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL V1m)))))) \Leftrightarrow (p (ap (ap c_2Arithmetic_2E_3C_3D \\
& \quad \quad V1m) V0n)))))))))
\end{aligned}$$

(60)

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0i \in ty_2Einteger_2Eint. \\ & (((p\ (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ (c_2Einteger_word_2EINT_MIN \\ & A.27a)\ (c_2Ebool_2Ethe_value\ A.27a)))\ V0i)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Eint_le \\ & V0i)\ (ap\ (c_2Einteger_word_2EINT_MAX\ A.27a)\ (c_2Ebool_2Ethe_value \\ & A.27a)))))) \Rightarrow ((ap\ (c_2Einteger_word_2Ew2i\ A.27a)\ (ap\ (c_2Einteger_word_2Ei2w \\ & A.27a)\ V0i)) = V0i)) \end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0v \in (ty_2Efc_2Ecart \\ & 2\ A.27a). (\forall V1w \in (ty_2Efc_2Ecart\ 2\ A.27a). (((ap\ (c_2Einteger_word_2Ew2i \\ & A.27a)\ V0v) = (ap\ (c_2Einteger_word_2Ew2i\ A.27a)\ V1w)) \Leftrightarrow (V0v = \\ & V1w)))) \end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\ & (ap\ (c_2Einteger_word_2EINT_MIN\ A.27a)\ (c_2Ebool_2Ethe_value \\ & A.27a)))\ (ap\ (c_2Einteger_word_2EINT_MAX\ A.27a)\ (c_2Ebool_2Ethe_value \\ & A.27a)))) \wedge ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ (c_2Einteger_word_2EINT_MAX \\ & A.27a)\ (c_2Ebool_2Ethe_value\ A.27a)))\ (ap\ (c_2Einteger_word_2EUINT_MAX \\ & A.27a)\ (c_2Ebool_2Ethe_value\ A.27a)))) \wedge (p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\ & (ap\ (c_2Einteger_word_2EUINT_MAX\ A.27a)\ (c_2Ebool_2Ethe_value \\ & A.27a)))\ (ap\ c_2Einteger_2Eint_of_num\ (ap\ (c_2Ewords_2Edimword \\ & A.27a)\ (c_2Ebool_2Ethe_value\ A.27a)))))) \end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow ((ap\ (c_2Einteger_word_2Ew2i \\ & A.27a)\ (c_2Ewords_2Eword_L\ A.27a)) = (ap\ (c_2Einteger_word_2EINT_MIN \\ & A.27a)\ (c_2Ebool_2Ethe_value\ A.27a))) \end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& (ap c_2Earithmetic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n)))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n))) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V0n) \\
& V1m)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap c_2Eprim_rec_2E_3C \\
& V1m) V0n)) (ap c_2Earithmetic_2ENUMERAL (ap (ap (ap c_2Enumeral_2EiSUB \\
& c_2Ebool_2ET) V0n) V1m))) c_2Enum_2E0))))
\end{aligned} \tag{67}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{68}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{69}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (71)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (75)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (76)$$

Theorem 1

$$\forall A_27a.nonempty \ A_27a \Rightarrow ((ap \ (c_2Einteger_word_2Ei2w \ A_27a) \ (ap \ (c_2Einteger_word_2EINT_MIN \ A_27a) \ (c_2Ebool_2Ethe_value \ A_27a))) = (c_2Ewords_2Eword_L \ A_27a))$$