

thm\_2Einteger\_word\_2Ei2w\_UINT\_MAX  
 (TMGX6RraaZA6QPooPZM91zJkYZGxo98EUQz)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c\_2Emin\_2E\_40 A) a)))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow & nonempty\ (ty\_2Epair\_2Eprod \\ & A0\ A1) \end{aligned} \quad (2)$$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \quad (3)$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint}) \quad (4)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) a)))$

**Definition 6** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint. (ap (c\_2Emin\_2E\_40 (ty\_2Einteger\_2Eint a)))$

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (5)$$

**Definition 7** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint. \lambda V1T2 \in ty\_2Einteger\_2Eint. (ap (c\_2Ebool\_2E_21 2) (\lambda V0t \in 2. V0t))$ .

**Definition 8** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E_21 2) (\lambda V0t \in 2. V0t))$ .

**Definition 9** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \Rightarrow p Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E_21 2)))$ .

**Definition 11** We define  $c\_2Einteger\_2Eint\_le$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint. \lambda V1y \in ty\_2Einteger\_2Eint. (c\_2Einteger\_2Etint\_mul (V0x V1y))$ .

Let  $c\_2Einteger\_2Etint\_mul : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_mul \in (((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (6)$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (7)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)}} \quad (8)$$

**Definition 12** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)$ .

**Definition 13** We define  $c\_2Einteger\_2Eint\_mul$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint. \lambda V1T2 \in ty\_2Einteger\_2Eint. (c\_2Einteger\_2Eint\_ABS (V0T1 V1T2))$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (9)$$

**Definition 14** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E_21 2) (\lambda V2t \in 2. (c\_2Ebool\_2E_21 2))))))$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (11)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (12)$$

**Definition 15** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num m)$ .

**Definition 16** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (c\_2Eprim\_rec\_2E\_3C (V0m V1n))$ .

**Definition 17** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c\_2Ebool\_2E\_21 2))(\lambda V2t \in$

**Definition 18** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 19** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let  $c\_2Einteger\_2Etint\_add : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_add \in (((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)}) \quad (14)$$

**Definition 20** We define  $c\_2Einteger\_2Eint\_add$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint.$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (15)$$

**Definition 21** We define  $c\_2Enum\_2E0$  to be  $(ap(c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)).$

**Definition 22** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap(ap(c\_2Earithmetic\_2Eint\_add$

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint)^{ty\_2Enum\_2Enum} \quad (16)$$

**Definition 23** We define  $c\_2Einteger\_2EEnum$  to be  $\lambda V0i \in ty\_2Einteger\_2Eint.(ap(c\_2Emin\_2E\_40 ty\_2Enum\_2Enum))$

**Definition 24** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 25** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0.$

**Definition 26** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap(ap(c\_2Earithmetic\_2Eint\_add$

**Definition 27** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let  $c\_2Earithmetic\_2EEEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (17)$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (18)$$

**Definition 28** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (19)$$

**Definition 29** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. \dots$

**Definition 30** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2Enum\_2Enum. \lambda V2m \in ty\_2Enum\_2Enum. \dots$

**Definition 31** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap$

Let  $ty\_2Efcp\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Efcp\_2Efinite\_image A0) \quad (20)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (21)$$

Let  $c\_2Ebool\_2Ethethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ebool\_2Ethethe\_value A\_27a \in (ty\_2Ebool\_2Eitself A\_27a) \quad (22)$$

Let  $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Efcp\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (23)$$

**Definition 32** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap c\_2Ebool\_2E\_2F\_5C$

**Definition 33** We define  $c\_2Efcp\_2Efinite\_index$  to be  $\lambda A\_27a : \iota. (ap (c\_2Emin\_2E\_40 (A\_27a^{ty\_2Enum\_2Enum}))$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efcp\_2Ecart A0 A1) \quad (24)$$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart A\_27a A\_27b \in ((A\_27a^{ty\_2Efcp\_2Efinite\_image A\_27b})^{(ty\_2Efcp\_2Ecart A\_27a A\_27b)}) \quad (25)$$

**Definition 34** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efcp\_2Ecart A\_27a A\_27b). (ap (c\_2Efcp\_2Edest\_cart A\_27a A\_27b) V0x))$

**Definition 35** We define  $c\_2Efcp\_2EFCP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (ap (c\_2Efcp\_2Edest\_cart A\_27a A\_27b) g)))$

**Definition 36** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_2Enum\_2Enum. (ap (c\_2Efcp\_2EFCP A\_27a) V0n))$

Let  $c\_2Einteger\_2Etint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Enum\_2Enum} \quad (26)$$

**Definition 37** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap\ c\_2Einteger\_2Eint\ V0T1)$

**Definition 38** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (ap\ (c\_2Ebool\ V0b)\ V1n)\ V0b)\ V1n)\ V0b)$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum)^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum} \quad (27)$$

**Definition 39** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ (ap\ (ap\ (c\_2Ebool\ V0w)\ A\_27a)\ A\_27a)\ A\_27a)\ A\_27a)$

Let  $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty}\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)})^{ty\_2Enum\_2Enum} \quad (28)$$

**Definition 40** We define  $c\_2Ewords\_2Eword\_2comp$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ (ap\ (ap\ (c\_2Ebool\ V0w)\ A\_27a)\ A\_27a)\ A\_27a)\ A\_27a)$

**Definition 41** We define  $c\_2Einteger\_word\_2Ei2w$  to be  $\lambda A\_27a : \iota.\lambda V0i \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap\ (ap\ (c\_2Ebool\ V0i)\ A\_27a)\ A\_27a)\ A\_27a)\ A\_27a)$

Let  $c\_2Einteger\_word\_2EUINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty}\ A\_27a \Rightarrow c\_2Einteger\_word\_2EUINT\_MAX\ A\_27a \in (ty\_2Einteger\_2Eint^{(ty\_2Ebool\_2Eitself\ A\_27a)})^{ty\_2Einteger\_2Eint} \quad (29)$$

**Definition 42** We define  $c\_2Ewords\_2Eword\_msb$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ (ap\ (ap\ (c\_2Ebool\ V0w)\ A\_27a)\ A\_27a)\ A\_27a)\ A\_27a)$

**Definition 43** We define  $c\_2Einteger\_word\_2Ew2i$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ (ap\ (ap\ (c\_2Ebool\ V0w)\ A\_27a)\ A\_27a)\ A\_27a)\ A\_27a)$

Let  $c\_2Ewords\_2EUINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty}\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)})^{ty\_2Enum\_2Enum} \quad (30)$$

**Definition 44** We define  $c\_2Ewords\_2Eword\_T$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Ewords\_2En2w\ A\_27a))$  (ap (c\_2Ewords\_2En2w A\_27a))

Assume the following.

$$\text{True} \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee (\neg(p \ V0t)))) \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0t \in 2.((\exists V1x \in \\ A\_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\ (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))) \end{aligned} \quad (37)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))))) \quad (38)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ True)) \quad (39)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (40)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ (p \ V0t))))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0t1 \in A\_27a.(\forall V1t2 \in \\ A\_27a.(((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2ET) \ V0t1) \\ V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2EF) \\ V0t1) \ V1t2) = V1t2)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\forall A \exists a. \text{nonempty } A \Rightarrow (\forall V0P \in (2^{A \rightarrow 27a}). ((\neg(\exists V1x \in A \exists a. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A \exists a. (\neg(p (ap V0P V2x))))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. \\ & ((p (ap (ap c\_2Einteger\_2Eint\_lt V0x) V1y)) \Leftrightarrow (p (ap (ap c\_2Einteger\_2Eint\_le \\ & (ap (ap c\_2Einteger\_2Eint\_add V0x) (ap c\_2Einteger\_2Eint\_of\_num \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) V1y)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. \\ & ((p (ap (ap c\_2Einteger\_2Eint\_le V0x) V1y)) \Leftrightarrow (p (ap (ap c\_2Einteger\_2Eint\_le \\ & (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) (ap (ap c\_2Einteger\_2Eint\_add \\ & V1y) (ap c\_2Einteger\_2Eint\_neg V0x)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0c \in ty\_2Einteger\_2Eint. (\forall V1x \in ty\_2Einteger\_2Eint. \\ & (\forall V2y \in ty\_2Einteger\_2Eint. ((p (ap (ap c\_2Einteger\_2Eint\_le \\ & (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) (ap (ap c\_2Einteger\_2Eint\_add \\ & V0c) V1x)) \Rightarrow ((p (ap (ap c\_2Einteger\_2Eint\_lt V2y) (ap c\_2Einteger\_2Eint\_neg \\ & V1x))) \Rightarrow ((p (ap (ap c\_2Einteger\_2Eint\_le (ap c\_2Einteger\_2Eint\_of\_num \\ & c\_2Enum\_2E0)) (ap (ap c\_2Einteger\_2Eint\_add (ap c\_2Einteger\_2Eint\_neg \\ & V0c) V2y)) \Leftrightarrow False))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0y \in ty\_2Einteger\_2Eint. (\forall V1x \in ty\_2Einteger\_2Eint. \\ & ((ap (ap c\_2Einteger\_2Eint\_add V1x) V0y) = (ap (ap c\_2Einteger\_2Eint\_add \\ & V0y) V1x)))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0z \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. \\ & (\forall V2x \in ty\_2Einteger\_2Eint. ((ap (ap c\_2Einteger\_2Eint\_add \\ & V2x) (ap (ap c\_2Einteger\_2Eint\_add V1y) V0z)) = (ap (ap c\_2Einteger\_2Eint\_add \\ & (ap (ap c\_2Einteger\_2Eint\_add V2x) V1y)) V0z)))))) \end{aligned} \quad (49)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint. ((ap (ap c_2Einteger_2Eint_add V0x) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) = V0x)) \quad (50)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint. ((ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x)) \quad (51)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_add V0x) V1y)) = (ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg V0x)) (ap c_2Einteger_2Eint_neg V1y))))) \quad (52)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_mul V0x) V1y)) = (ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_neg V0x)) V1y)))) \quad (53)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_mul V0x) V1y)) = (ap (ap c_2Einteger_2Eint_mul V0x) (ap c_2Einteger_2Eint_neg V1y))))) \quad (54)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint. ((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_neg V0x)) = V0x)) \quad (55)$$

Assume the following.

$$((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \quad (56)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((ap c_2Einteger_2Eint_of_num V0m) = (ap c_2Einteger_2Eint_of_num V1n)) \Leftrightarrow (V0m = V1n))) \quad (57)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap c_2Einteger_2Enum (ap c_2Einteger_2Eint_of_num V0n)) = V0n)) \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in ty\_2Einteger\_2Eint. (\forall V1n \in ty\_2Enum\_2Enum. \\ & (\forall V2m \in ty\_2Enum\_2Enum. (((ap (ap c_2Einteger_2Eint_add \\ & (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0p) = V0p) \wedge ((( \\ & ap (ap c_2Einteger_2Eint_add V0p) (ap c_2Einteger_2Eint_of_num \\ & c_2Enum_2E0)) = V0p) \wedge (((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\ & c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \wedge \\ & (((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_neg V0p)) = \\ & V0p) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\ & (ap c_2Earithmetic_2ENUMERAL V1n))) (ap c_2Einteger_2Eint_of_num \\ & (ap c_2Earithmetic_2ENUMERAL V2m))) = (ap c_2Einteger_2Eint_of_num \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V2m)))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\ & (ap c_2Earithmetic_2ENUMERAL V1n))) (ap c_2Einteger_2Eint_neg \\ & (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\ & V2m))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap \\ & (ap c_2Earithmetic_2E_3C_3D V2m) V1n)) (ap c_2Einteger_2Eint_of_num \\ & (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V1n) \\ & V2m))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\ & (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V2m) \\ & V1n)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\ & (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\ & V1n)))) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\ & V2m))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap \\ & (ap c_2Earithmetic_2E_3C_3D V1n) V2m)) (ap c_2Einteger_2Eint_of_num \\ & (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V2m) \\ & V1n)))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\ & (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V1n) \\ & V2m)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\ & (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\ & V1n)))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\ & (ap c_2Earithmetic_2ENUMERAL V2m)))) = (ap c_2Einteger_2Eint_neg \\ & (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B V1n) V2m)))))))))))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. (\forall V0m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 V0n)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Einteger_2Eint_lt \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V0n)))) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow \\
& False) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V0n)))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Einteger_2Eint_lt \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V0n))) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 V0n)))) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap \\
& c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V0n)))) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap \\
& c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL V0n))) \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap \\
& c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V0n)))) \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1m))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V0n)))) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V1m))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Einteger_2Eint_lt \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V0n))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V1m))) \Leftrightarrow False) \wedge ((p (ap (ap c_2Einteger_2Eint_lt \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V0n))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V1m)))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))))))) \\
& (60)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (((ap c_2Einteger_2Eint_of_num V0m) = (ap c_2Einteger_2Eint_of_num \\
& V1n)) \Leftrightarrow (V0m = V1n))) \wedge (\forall V2x \in ty\_2Einteger\_2Eint. (\forall V3y \in \\
& ty\_2Einteger\_2Eint. (((ap c_2Einteger_2Eint_neg V2x) = (ap c_2Einteger_2Eint_neg \\
& V3y)) \Leftrightarrow (V2x = V3y)))) \wedge (\forall V4n \in ty\_2Enum\_2Enum. (\forall V5m \in \\
& ty\_2Enum\_2Enum. (((ap c_2Einteger_2Eint_of_num V4n) = (ap c_2Einteger_2Eint_of_num \\
& V5m)) \Leftrightarrow ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))) \wedge \\
& ((ap c_2Einteger_2Eint_of_num V4n) = (ap c_2Einteger_2Eint_of_num V5m)) \Leftrightarrow \\
& ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))))))) \\
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0v \in (ty\_2Efcp\_2Ecart \\
& 2 A\_27a). (\forall V1w \in (ty\_2Efcp\_2Ecart 2 A\_27a). (((ap (c_2Einteger\_word\_2Ew2i \\
& A\_27a) V0v) = (ap (c_2Einteger\_word\_2Ew2i A\_27a) V1w)) \Leftrightarrow (V0v = \\
& V1w)))) \\
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty A\_27a \Rightarrow ((ap (c_2Einteger\_word\_2EUINT\_MAX \\
& A\_27a) (c_2Ebool\_2Ethel\_value A\_27a)) = (ap c_2Einteger_2Eint_of_num \\
& (ap (c_2Ewords\_2EUINT\_MAX A\_27a) (c_2Ebool\_2Ethel\_value A\_27a)))) \\
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty A\_27a \Rightarrow (p (ap (ap c_2Einteger_2Eint_lt \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (c_2Einteger\_word\_2EUINT\_MAX \\
& A\_27a) (c_2Ebool\_2Ethel\_value A\_27a)))) \\
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty A\_27a \Rightarrow ((ap (c_2Einteger\_word\_2Ew2i \\
& A\_27a) (c_2Ewords\_2Eword\_T A\_27a)) = (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic\_2ENUMERAL \\
& (ap c_2Earithmetic\_2EBIT1 c_2Earithmetic\_2EZERO)))))) \\
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty A\_27a \Rightarrow ((ap (c_2Einteger\_word\_2Ew2i \\
& A\_27a) (ap (c_2Ewords\_2Eword\_2comp A\_27a) (ap (c_2Ewords\_2En2w \\
& A\_27a) (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT1 \\
& c_2Earithmetic\_2EZERO)))))) = (ap c_2Einteger_2Eint_neg (ap \\
& c_2Einteger_2Eint_of_num (ap c_2Earithmetic\_2ENUMERAL (ap \\
& c_2Earithmetic\_2EBIT1 c_2Earithmetic\_2EZERO)))))) \\
\end{aligned} \tag{66}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (71)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V1q) \vee (p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q) \vee (\neg(p V0p)))))))))))))) \end{aligned} \quad (72)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q) \vee (p V2r)))) \wedge (((p V1q) \vee ((\neg(p V0p) \wedge (p V2r)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (73)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q) \vee (p V2r)))) \wedge (((p V0p) \vee ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (74)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q) \vee (p V0p))))))) \quad (75)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow ((ap(c\_2Ewords\_2Eword\_2comp \\ & A\_27a) (ap(c\_2Ewords\_2En2w A\_27a) (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) = (c\_2Ewords\_2Eword\_T \\ & A\_27a)) \end{aligned} \quad (76)$$

**Theorem 1**

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow ((\text{ap } (\text{c\_2Einteger\_word\_2Ei2w } A_27a) (\text{ap } (\text{c\_2Einteger\_word\_2EUINT\_MAX } A_27a) (\text{c\_2Ebool\_2Eth\_value } A_27a))) = (\text{c\_2Ewords\_2Eword\_T } A_27a))$$